DESIGN AND VALIDATION OF AN ACTIVE TRAILER STEERING CONTROLLER FOR LONGER COMBINATION VEHICLES USING A HARDWARE-IN-THE-LOOP PLATFORM



A. RAHIMI
National Research Council Canada.

N. YOVCHEV
National Research Council Canada.

W. HUANG
National Research Council Canada.

Abstract

Articulated heavy vehicles (AHVs), especially longer combination vehicles (LCVs), are essential for long-distance freight transport but face stability and maneuverability challenges. To address these, we validate an active trailer steering (ATS) control system based on the linear quadratic regulator (LQR) method to control trailer axle steering in an A-double LCV. A five-degree-of-freedom (5-DOF) yaw-plane linear model with steerable axles was developed, with key parameters updated online to match a high-fidelity, non-linear TruckSim model. Offline, particle swarm optimization (PSO) was used to generate look-up tables by minimizing differences between the linear model and TruckSim states, based on open-loop step-steering maneuvers. LQR weighting factors were similarly optimized to reduce trailing units' off-tracking with respect to the tractors path. The optimal weights were used then to make a look-up table to generate real-time simulation trailer optimal steering based on the vehicle speed. A hardware-in-the-loop (HIL) platform with real-time simulation and CAN communication was employed to validate the controller in various scenarios. Results confirm the ATS system's potential to enhance AHV stability, maneuverability, and safety, paving the way for more efficient transportation.

Keywords: Articulated Heavy Vehicles (AHVs), Longer Combination Vehicles (LCVs), A-Double, Active Trailer Steering (ATS), Modified LQR, Hybrid Linear Model.

1. Introduction

Canada has increasingly turned to using articulated heavy vehicles (AHVs) and longer combination vehicles (LCVs) for the transportation of goods over the last few decades. When compared to single-unit vehicles or tractor-semi-trailers, LCVs can increase cargo capacity significantly per driver, thereby increasing efficiency and reducing emissions during transportation. Since their introduction in Canada, many studies such as Billing and Madill (2019) have demonstrated their necessity in transporting resources across provincial borders, while also addressing costs, emissions and infrastructure benefits.

While AHVs are beneficial in this sector, they perform poorly in maneuverability tests due to their complex and large structures, and impose severe traffic accidents resulting in fatal injuries and financial costs in North America (Rahimi and He, 2020). Huang et al. (2023) discussed how LCVs, which can reach up to 40m in length in Canada, exhibit a multitude of challenges with their maneuverability and lateral stability at low- and high-speeds; respectively. This can lead to rollover, jack-knifing, high rearward amplification, and poor path-following off-tracking. Trailer manufacturers have begun implementing trailer steering systems based on the articulation of the trailer compared to the tractor. These passive and semi-active trailer steering systems show improvements of trailer stability for low-speed driving scenarios but typically perform worse than un-steered trailers for higher-speed maneuvers, even under typical road conditions (Jujnovich and Cebon, 2002).

Many of the limitations of passive and semi-active trailer steering system can be overcome by active trailer steering (ATS) controllers, where driver input and tractor-trailer geometry are both considered (Jujnovich and Cebon, 2002). ATS controllers which have been designed for tractor-semi-trailers have shown improved tail swing, path following, lateral stability, and maneuverability for various curves and speeds compared to un-steered and passive methods (Huang, Rahimi, Yu and Steiginga, 2021). The downside to some of these control strategies is often in their complexity, both mechanically and computationally, which is only amplified by the addition of more trailers. ATS control strategies for a tractor/semi-trailer using a LQR controller have been proposed to improve the vehicle’s performance for low- and high-speed maneuvers (Kim et al., 2016). Sikder’s thesis (2017) also studies the applications of LQR control strategies for B-train doubles, and discusses some of the limitations of using a basic LQR strategy in practical applications.

Hardware-in-the-loop (HIL) systems offer a method of validating the computational requirements for ATS controllers on LCVs without the need for expensive road-test facilities. A HIL platform can run mathematical controller models with information sensed in real-time, with higher timing constraints compared to software simulations. Additionally, the added hardware allows for real propagation delays, which further constrain the requirements for a functional controller.

This paper discusses a modified design and validation of a LQR controller for an A-train double using a so-called hybrid linear model which combines analytical and data-driven method (a pseudo-black-box approach). The control strategy is analyzed with co-simulations using a HIL platform. Its contributions not only validate the efficacy of this ATS controller in LCVs with three articulation points, but also demonstrate its potential in real-time using HIL co-simulations and highlights the application of a linear-model-based strategy for various conditions, including low-speed maneuvers.

2. Vehicle Dynamics Model

An A-train double, which contains a conventional tractor and two trailers joined together by a conventional dolly, was chosen as the subject of this control strategy. The tractor has three axles, of which only the front is steered. The trailers have tridem axle group, where the last two axles are steerable. A schematic representation of linear 5-DOF single-track tractor-trailer model can be seen in Figure. 1.

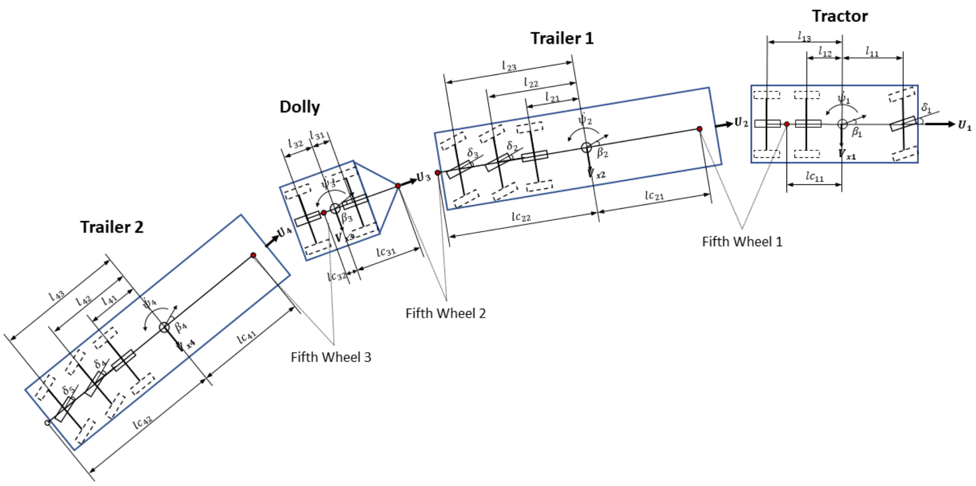

Figure 1 – 5-DOF A-train double model.

The vehicle dynamics equations of motion for this vehicle combination and the relevant simplifying assumptions are detailed in Rahimi's thesis (2023). Table 1 shows the notations found in this paper.

Table 1 - Notations

Symbol	Definition	Symbol	Definition
β_i	Side-slip angle of vehicle body *i*	m_i	Sprung mass of vehicle body *i*
$\dot{\psi}_i$	Yaw rate of vehicle body *i*	I_{izz}	Yaw moment of inertia of sprung mass of vehicle body *i*
U_i	Longitudinal speed of vehicle body *i*	V_{yi}	Lateral speed of vehicle body *i*
δ_1	Steer angle of tractor's front axle	A_{yi}	Lateral acceleration of vehicle body *i*
δ_2	Steer angle of 1st trailer second axle	C_i	Tire cornering stiffness for tractor, 1st trailer, dolly and 2nd trailer axle groups in order. Note that tractor has two axle groups.
δ_3	Steer angle of 1st trailer third axle	q_i	Q matrix elements
δ_4	Steer angle of 2nd trailer second axle	L_{ijc}	Distance between center of gravity of vehicle *i* and *j* fifth wheel
δ_5	Steer angle of 2nd trailer third axle	L_{ij}	Distance between center of gravity of vehicle *i* and its *j*th axle

3. LQR Control Strategy

The LQR control strategy aims to drive the system states toward an optimal trajectory by minimizing a cost function that balances state deviations and control effort. It achieves this through a set of optimal feedback gains computed from a weighted quadratic performance index. (Sikder, 2017). The cost function of the LQR controller in the general form for a state-space model is

$$J(u) = \int_0^{\infty} (x^T Q x + u^T R u + 2x^T N u)\, dt \quad (1)$$

where u represents the system inputs, x is matrix of the states, and Q, R, and N are matricies of weighting factors tuned by the controller's designer. For this system, the cost function relates to the states β_{i} and $\dot{\psi}_i$, which are used to describe the heading and articulation of each vehicle unit. Steering the trailers can bring these states to a desired value, minimizing effects such as off-tracking, tail-swinging, and lateral acceleration. To accomplish this, the cost function in the current study is written such that

$$\begin{aligned} J = \sum\big(&q_1(\beta_1 - \beta_{1d})^2 + q_2(\dot{\psi}_1 - \dot{\psi}_{1d})^2 + q_3(\beta_2 - \beta_{2d})^2 + q_4(\dot{\psi}_2 - \dot{\psi}_{2d})^2 \\ &+ q_5(\beta_3 - \beta_{3d})^2 + q_6(\dot{\psi}_3 - \dot{\psi}_{3d})^2 + q_7(\beta_4 - \beta_{4d})^2 + q_8\big(\dot{\psi}_4 - \dot{\psi}_{4d}\big)^2 \\ &+ r_1\delta_2^2 + r_2\delta_3^2 + r_3\delta_4^2 + r_4\delta_5^2\big) \end{aligned} \quad (2)$$

where $q_1 - q_8$ and $r_1 - r_4$ are the weighting matrices' elements, and states with the subscript 'd' are the desired states calculated by the state-space model. The output steering angles for the trailers are calculated by the state feedback equation

$$u = -Ke \quad (3)$$

where $e = x - x_d$ is the error between the current and desired states and $u = [\delta_2 \quad \delta_3 \quad \delta_4 \quad \delta_5\,]^T$. K is the feedback matrix derived from the solution S of the Riccati Equation using, such that,

$$A^T S + SA - (SB + N)R^{-1}(B^T S + N^T) + Q = 0 \quad (4)$$

and

$$K = R^{-1}(B^T S + N^T) \quad (5)$$

Figure 2 shows an overview schematic of the modified LQR controller structure. This controller considers a data-driven hybrid model of the vehicle dynamics based on the linear 5-DOF model, and an adaptive LQR approach. The parameters of both the hybrid linear model and the modified LQR controller are updated on-line based on sensor information from the plant.

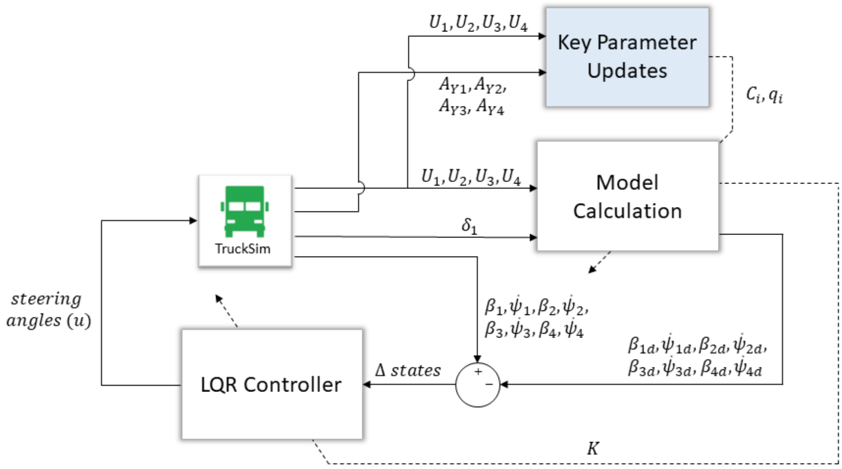

Figure 2 – Modified LQR controller structure schematic.

3.1 Weighted Simulated Annealing Particle Swarm Optimization for Model Parameters and Controller Tuning

Particle swarm algorithms are commonly used to optimize parameters, since it is efficient in globally minimizing cost functions with minimal iterations. The method was conceptualized in part to mimic the behaviors of swarming fish and bird groups, and thus has similarities to other genetic algorithms (Kennedy and Eberhart, 1995). The implementation of additional simulated annealing method discussed by Gao and Xie (2004) allows for small mutations in each particle population for additional optimization. To further improve the performance of the simulated annealing particle swarm optimization (SA-PSO) algorithm, an adaptive inertial weight strategy from Karthick et al. (2016) was used to dynamically change the particle speeds at each generation depending on their success rates.

Both the cornering stiffness values of the wheels used for the hybrid linear model and the Q and R values of the LQR controller can be determined using the follow algorithm:

1. Initialize the particle swarm and range for inertia weights, velocities, speeds, and particle positions.
2. For each generation, the particles will update their position based on their current headings.
3. For each generation, the new particle positions are used as the parameters for the model, wherein the value of the cost function is determined.
4. For each generation, the cost function values of each particle are used to update their inertial weights, as well as the generational and global best potions.
5. The position of the particle that yielded the lowest cost using the defined cost function is returned once all the generations are complete.

The cost function for the weighted adaptive SA-PSO tries to minimize the states from the model compared to some reference for both the optimization of the cornering stiffness values of the wheels and for the LQR parameters. This is explained in detail in the following sections.

3.1.1 Cornering Stiffness Optimization

As seen in the vehicle dynamics model equations, the cornering stiffness is a key parameter to the model. In this study, it is assumed that the relationship between the tire lateral force and the wheel slip angle is linear, and is given by a constant value i.e. the cornering stiffness for each axle of the vehicle. To improve the similarity between the linear model and the TruckSim non-linear model in terms of the yaw-rate and sideslip states, it is assumed that the parameter can be updated depending on the current lateral acceleration and speed of each vehicle unit, individually. A 3D look-up table can be created off-line by giving the same steering and vehicle speed inputs to the linear model and TruckSim model and minimizing the cost function given in equation (6). The look-up table is used then to update the parameters of the model in real time to ensure the model agrees with the dynamics of the high-fidelity A-train double model.

The cost function for the cornering stiffness optimization is

$$F = \sum_{j=1}^{4} \sum_{i=1}^{N} \left[\beta_{jT}(i) - \beta_{jM}(i)\right]^2 + \left[\dot{\psi}_{jT}(i) - \dot{\psi}_{jM}(i)\right]^2 \qquad (6)$$

where $\beta_{jT}(i)$ and $\dot{\psi}_{jT}(i)$ are the side-slip angles and yaw rates for each vehicle unit measured in TruckSim for an open-loop step steering maneuver, and $\beta_{jM}(i)$ and $\dot{\psi}_{jM}(i)$ are the side-slip angles and yaw rates for each vehicle unit from the model for the same open-loop step steering maneuver. N is the number of samples taken at constant time intervals during the simulations.Each open-loop maneuver was designed to cover a range of speeds from 10km/h to 100km/h, with each speed iterating through a set of step steering inputs that would generate a variety of lateral accelerations on the units. By measuring the vehichle units' lateral acceleration and speed, these can act as the two sensor outputs used as inputs to the look-up table for the optimal cornering stiffness values. Figure 3 shows the resulting parameter values for the two trailers as an example. For the sake of brevity, additional graphs have been omitted.

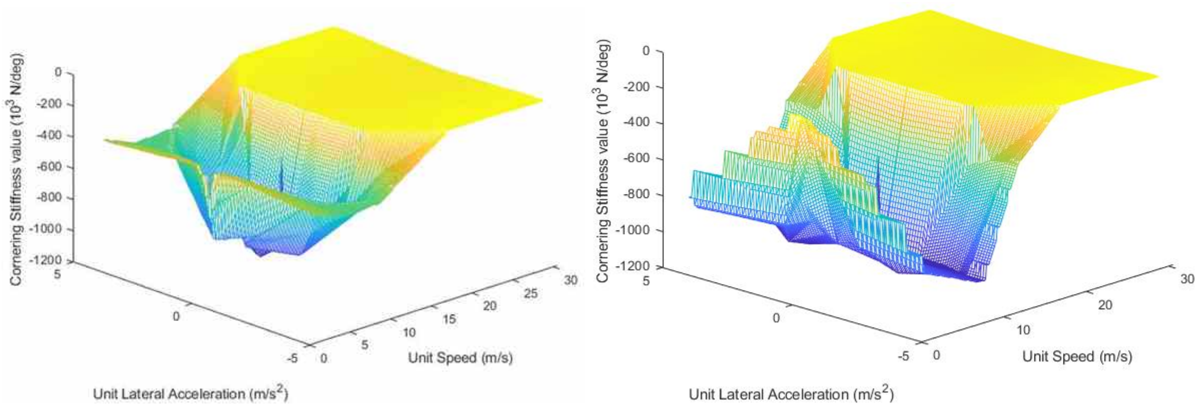

Figure 3 – Cornering stiffness for the first (left) and the second (right) trailer's axles.

Using the cornering stiffness look-up table, the model was then validated using TruckSim by sweeping over various speeds with a sinusoidal steering input that is speed ranges from 100 to 0 km/h, with steering wheel angle of 60*sin(t) where t is the simulation time. Figures 4 shows that the yaw rates and lateral accelerations are agreeable between the hybrid linear model and the TruckSim reference for the same maneuver.

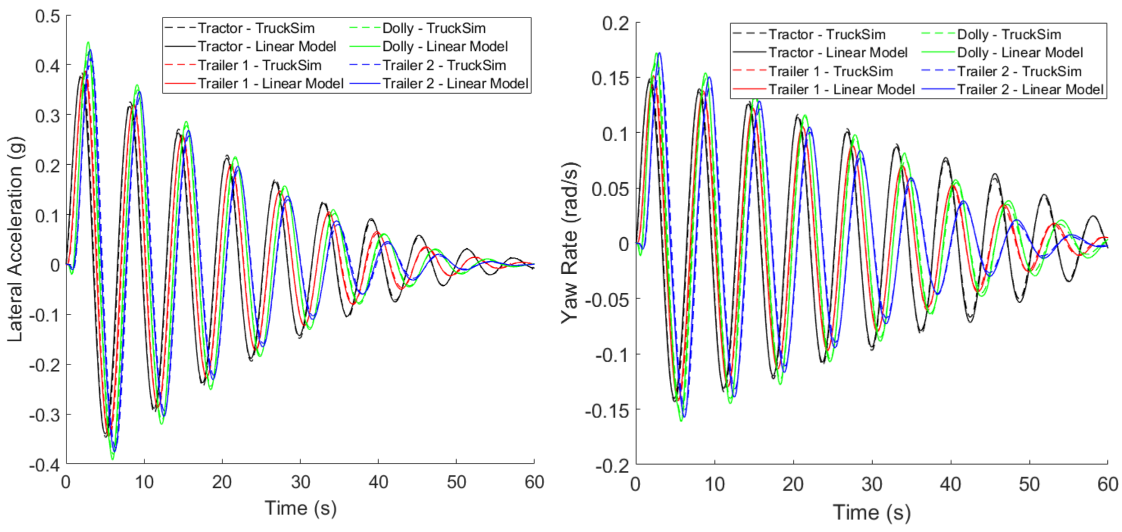

Figure 4 – Vehicle units' lateral accelerations (left) and yaw rates (right).

3.1.2 LQR Parameters

One of the fundamental parameters of the LQR controller, the Q weights, determine the impact of the states on the cost function of the LQR. To ensure the robustness of the controller for varying speeds and maneuvers, as similar optimization to the cornering stiffnesses was performed in order to create a look-up table of weights. These Q weights will be used to update the gain to the system depending on the speed and lateral acceleration measured in the tractor.

The goal of the optimization was to minimize the path-following off-tracking of the vehicle units for a combination of step steering and vehicle speed values. Off-tracking is defined in this paper as the difference between the path followed by the tractor and that of its trailing units. The optimization problem variables, q_i weights, were used to generate unique trailer steering configurations for each iteration of the optimization. To achieve this, the states at 10 positions along the path were evaluated for all vehicle units using the hybrid linear model. The cost function for the optimization problem was defined as the summation of errors across these 10 positions. The optimal values for q_i were then stored for that specific combination of step steering and vehicle speed. This process was repeated across several speeds and step steering inputs, and the optimal values from all simulations were stored in a look-up table for online weights generation. The cost function is

$$F = E_1 + E_2 \tag{7}$$

where

$$E_1 = \sum_{i=1}^{\mathrm{N}} abs(\beta_1(i) - \beta_2(i)) + abs\left(\dot{\psi}_1(i) - \dot{\psi}_2(i)\right) \quad (8)$$

$$E_2 = \sum_{i=1}^{\mathrm{N}} abs(\beta_1(i) - \beta_4(i)) + abs\left(\dot{\psi}_1(i) - \dot{\psi}_4(i)\right) \quad (9)$$

and $\beta_j(i)$ and $\dot{\psi}_j(i)$ are the side-slip angles and yaw rates for the respective vehicle units for an open-loop step steering. N is the number of samples taken at constant time intervals during the simulation. The cost function was able to find optimal Q weights which paired with the constant R weights of $r_1 = r_2 = 0.4$, $r_3 = r_4 = 0.3$, resulted in minimal error in the states between the tractor and the trailers, thereby reducing the path-following off-tracking.

4. Simulation and Discussion

The validation of the control strategy was done using a co-simulation with TruckSim, which is able to accurately represent a real-world vehicle, and MATLAB/Simulink. Four maneuvers were tested with the use of a closed-loop driver model in TruckSim, which adjusts the tractor speed and the steering wheel angle, braking, and throttle according to the path. Table 2 shows the values of the vehicle parameters used in TruckSim.

Table 2 – Values for vehicle system parameters

Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
m_1(kg)	6310	I_{2zz}(kg·m^2)	246000	l_{22c}(m)	8.240	l_{12}(m)	3.115	l_{31}(m)	0.565
m_2(kg)	10850	I_{3zz}(kg·m^2)	375	l_{32c}(m)	2.065	l_{13}(m)	4.465	l_{32}(m)	0.723
m_3(kg)	2397.2	I_{4zz}(kg·m^2)	246000	l_{33c}(m)	0.080	l_{21}(m)	3.218	l_{41}(m)	3.218
m_4(kg)	10850	l_{c11}(m)	3.218	l_{43c}(m)	6.760	l_{22}(m)	5.048	l_{42}(m)	5.048
I_{1zz}(kg·m^2)	19665	l_{21c}(m)	6.760	l_{11}(m)	2.145	l_{23}(m)	6.878	l_{43}(m)	6.878

4.1 Simulation of 90-degree Turn and Roundabout Low Speed

In low-speed scenarios, it's crucial for the trailing units of a vehicle to follow the same path as the lead unit, especially in densely populated areas or roads with many obstacles. Drivers of LCVs must anticipate the motion of their trailing units to manage transient off-tracking during sharp turns. Figures 5 and 6 show the results of a 90-degree turn and a round-about maneuver at 20km/h. Both Figures depict the off-tracking for un-steered, command-steered and active-steered trailers, as well as the trailer axle steering angles.

In a simulation of a 90-degree turn with a 15m radius, the second trailer's swept path width was reduced by 45% compared to a conventional trailer (around 2.3m improvement). Similarly, in a roundabout scenario with a 15m radius at the same speed, the maximum off-tracking was reduced by nearly 75% (3.87m improvement).

Command steering, a passive trailer steering method, improves low-speed maneuverability by using the articulation at the fifth wheel to steer the axles. However, active trailer steering controllers can achieve even better results. In both scenarios, actively-steered trailers outperformed those with command steering. The ATS controller was able to improve the swept path width by 14% and 55% compared to the command-steered trailers for the 90-degree turn and the roundabout respectively.

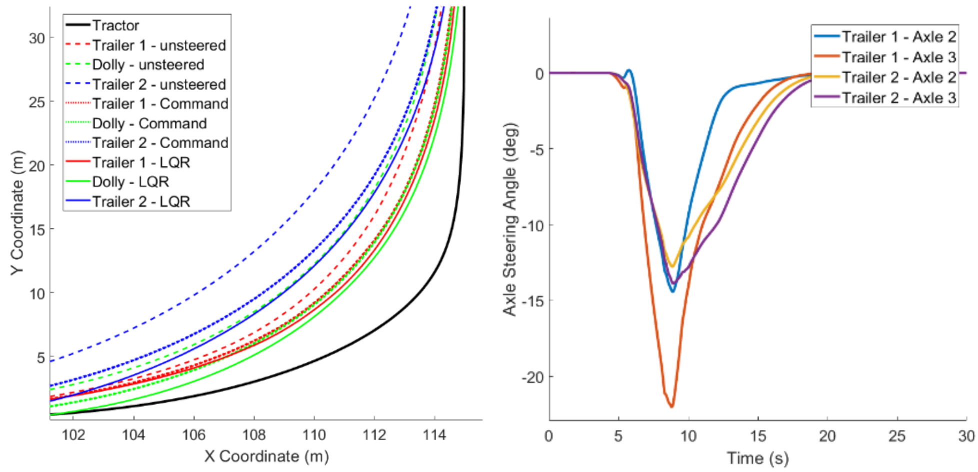

Figure 5 – Vehicle units' path (left) and axle steering inputs (right) for 90-degree turn.

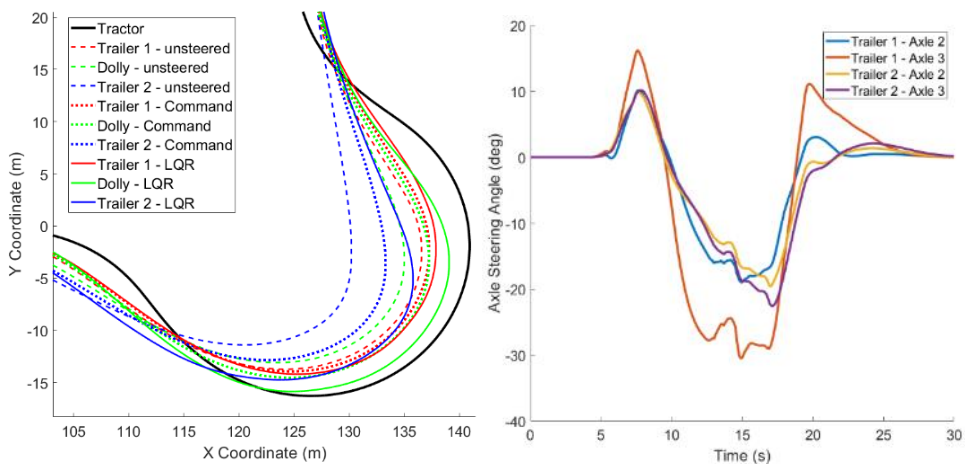

Figure 6 – Vehicle units' path (left) and axle steering inputs (right) for roundabout.

4.2 Simulation of Double Lane-change and Long-radius Curve at High Speed

A double lane-change (DLC) maneuver is a common high-speed evasive action where stability and roll-over risk are major concerns. Performance is often measured by rearward amplification, the ratio of the last trailing unit's lateral acceleration to that of the tractor. As the command-steered trailers show worse high-speed performance and lateral instability compared to non-steered trailers, the axles are locked above a speed threshold. Although not optimized for rearward amplification, the adaptive LQR controller showed a slight reduction in rearward amplification compared to conventional trailers which makes the controller useful for all driving scenarios unlike the command-steered controller. This is achieved because it is assumed that the trailers are steered in a way to follow the path travelled by tractor not the road reference path which is followed by tractor, thereby improving the lateral stability at high-speeds (Rahimi, 2023). Figure 7 illustrates the vehicle units' path during a long-radius curve at 100 km/h at the apex, and the lateral accelerations during a DLC maneuver at the same speed.

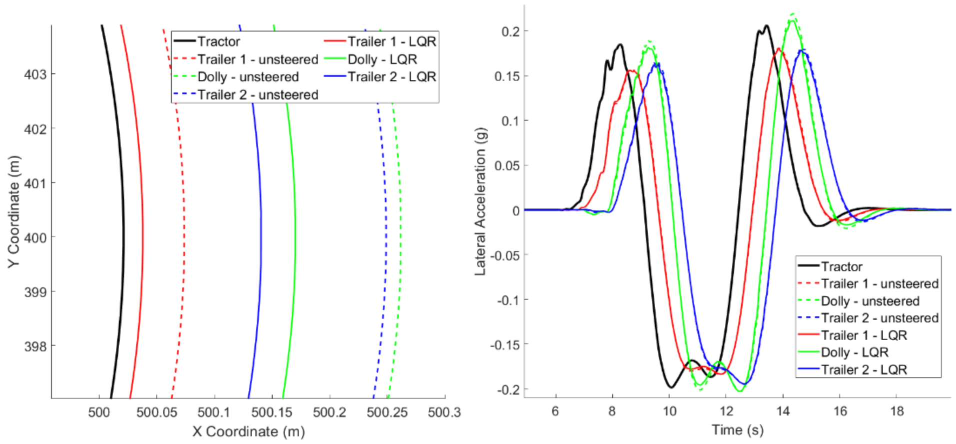

Figure 7 – Vehicle units' path for curve (left) and lateral accelerations for DLC (right).

Steady-state off-tracking is a performance measure for high-speed yet non-evasive maneuvers for AHVs. A high-speed long-radius curve maneuver, requires minimizing steady-state off-tracking to prevent trailers from straying from their lanes. In a simulation of a 400m radius curve at 100 km/h, the off-tracking at the apex was reduced by about 40% (around 25cm) for the actively steered second trailer compared to the conventional.

5. Hardware-in-the-Loop Validation

HIL co-simulations validate controller design efficacy while considering real-time system and hardware constraints, such as propagation delays, computational power, and noise. By using an HIL simulator and a software platform like TruckSim, ATS control strategies can be tested for hardware parameters in a safer and more cost-effective manner than road tests during the initial design stages.

The HIL platform consists of two OPAL-RT targets communicating via CAN protocol, commonly used in the automotive sector. These targets connect through Ethernet to a host computer running RT-Lab, which co-simulates with MATLAB/Simulink and TruckSim. One target behaves as the simulator, collecting real-time "sensor data" from TruckSim, while the other serves as the electronic control unit (ECU) with the modified LQR controller. Figure 8 shows the HIL setup.

To ensure that the controller could work in real-world scenarios, it was designed to have a limited number of sensors, requiring only the yaw rates and longitudinal/lateral accelerations from each unit to estimate the states and update the parameters of the model. The frequency at which the axles could recieve was limited to 100ms, which is fast enough to create a smooth response in the system, but not to fast as to overload the CAN connection. Figure 9 shows the execution time needed to simulate the controller for the entire complex route.

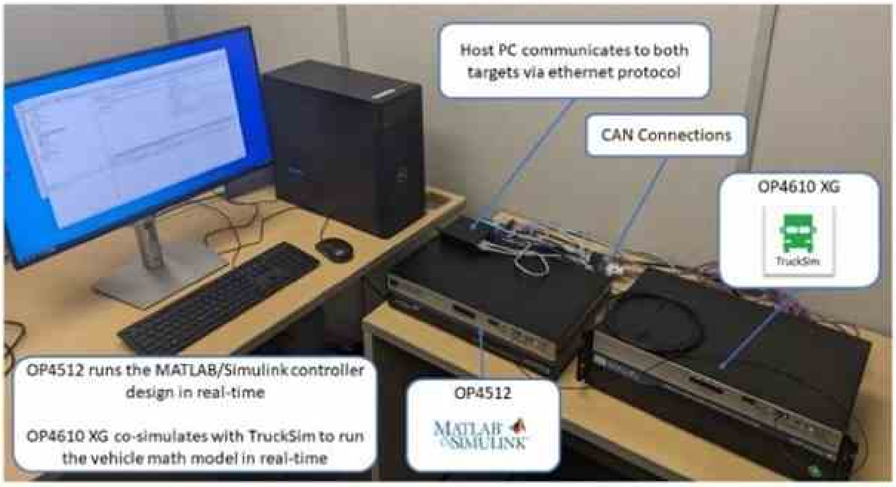

Figure 8 – HIL platform containing the host PC and two real-time target machines.

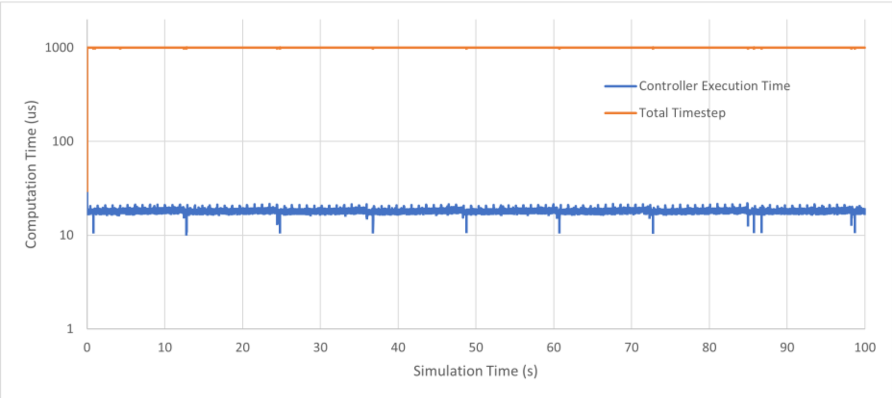

Figure 9 – Execution time compared to total step size of real-time simulation

As it is seen from the graph, the execution time never exeeded 2.5% of the total 1ms timestep. The controller was able to achieve the same improvements during real-time for the complex route containing all the previously mentioned low- and high-speed maneuvers, while respecting the target's real-time hardware limitations.

6. Controller Robustness Analysis Under Varying Conditions

The controller performance was examined for various road friction and payload conditions. Different frictions of the road were considered for low-speed 90-degree turn and high-speed DLC Maneuvers. Table 3 demonstrates the percentage of improvement in the selected criteria compared to command steering for low speeds, and the un-steered trailer for high-speed scenarios. For each maneuver, the speed is kept the same during the test, that is 20 km/h for low speed and 85 km/h for the DLC.

As seen by the results, the controller which was optimized for typical road conditions (friction coefficient above 0.8) outperforms the conventional un-steered and command-steered trailers in simulations of wet surfaces (coefficient below 0.5). Further testing is needed for snow and icy conditions (coefficient below 0.3) since the speeds chosen for these tests produced varying results. This is expected since for icy roads, the speed, severity of the maneuver, and the payload have a larger impact. Table 4 demonstrates test results when only the payload of the trailers varied.

Table 3 – LQR Performance improvement of steered trailers under various road C.O.F.

Road Friction (μ)	Low-speed (swept path width)	High-speed (rearward amplification)
0.8	9.8%	3%
0.6	10.5%	3.1%
0.4	16.2%	4.2%

Table 4 – LQR Performance improvement of steered trailers with different payloads

Trailer payload (kg/trailer)	Low-speed (swept path width)	High-speed (rearward amplification)
0	2.1%	2.9%
9,000	11.2%	4.1%
18,000	13.2%	6.3%

Even though the controller was not optimized to minimize rearward amplification, in both trials under the worst conditions i.e., the maximum payload and least road friction, the LQR controller improved the performance of the trailers compared to un-steered and command steering units.

7. Conclusions

An active trailer steering strategy for an A-train double was developed using an adaptive LQR controller. The controller was optimized to reduce the off-tracking of the following units in the LCV compared to the target path of the tractor, while still respecting the limitations of a physical vehicle. Conclusions about the practical use of ATS systems, as well as the implementation of LQR control strategies using a hybrid linear model can be drawn:

1. Active trailer steering using the proposed modified LQR controller improves performance at both low and high speeds for an A-train double without needing additional high-level control strategies, or deactivating the controller for high-speed maneuvers like command-steered trailers. By optimizing the controller for various scenarios and dynamically adjusting weighting parameters based on vehicle states, it reduces off-tracking at low and high speed, while maintaining the lateral stability and even improve it slightly compared with conventional vehicle. Further optimization where rearward amplification is considered in the cost function and the addition of weight factors based on the speed of the vehicle can result in an even more comprehensive controller.
2. The controller was able to run through a co-simulation with a real-time target, and was able to run without producing any overruns. The CAN cables were able to simulate real-time data transfers between sensors and the electronic control unit. Future work will include hydraulic and mechanical delays within the system imposed by the electro-

hydraulic steering system in addition to the current propagation delays, to better simulate the feasibility of the controller. To achieve this, additional hardware i.e., the trailer steering mechanism will be added to the HIL simulator.

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