An Eigenvalue Study of Articulated Heavy Vehicles with Central Axle Trailers

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Abstract
Recently, central axle trailers have a wide use in China, especially after issuing a national standard named GB1589-2016. But there happened more or less traffic accidents when driven them. In order to improve the stability of central axle trailers, the paper established the theoretical dynamic model of vehicle-trailer combination, and analyse its system eigenvalues.The influence of different trailer parameters on the system high-speed stability was examined. The results showed that its critical speed were only 20 m/s for a given example of central axle trailer. It is unstable when driven on the express road because the forward velocity is usually above the critical speed. It also found that the dominant structural parameters affected the stability, which were the trailer mass distribution, trailer axle positions, and trailer length. The mass distribution is the most important influence factor on the stability, while the trailer length has less significant effect. So the suggestion is the CG (Center of gravity) of load should be in the front of trailer axle to enhance the stability .
Keywords: Eigenvalue, Central Axle Trailer, Dynamic

1. INTRODUCTION

After issuing the national standard, named GB1589-2016 “Limits of dimensions, axle load and masses for motor vehicles, trailers and combination vehicles” , central axle trailers are generally developed and widely produced in China. Central axle vehicle combination comprised two parts: traction vehicle and trailer, as is described in Figure 1. The trailer, whose axle is positioned at the trailer’s center, is a form of a full trailer. Since its load is also located in the center of trailer’s gravity, only a small vertical load is applied to the traction vehicle when the vehicle is working.

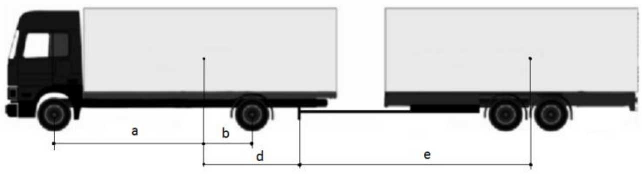


Fig1 : Side View for Central Axle Vehicle

A central axle trailer cannot move vertically relative to the tractor because it is connected with the tractor by a connecting device, also named a coupler, a kind of master and slave hook. Compared with the semi-trailer, the semi-trailer connects to the tractor by a pin plate and the saddle structure which can avoid sway and swing, while the coupler cannot prevent the center axle trailer from sway and swing. Therefore, the central axle trailer has a poorer stability than the semi-trailer. In order to reduce the risk of sway or swing and improve the stability of central axle trailer, the theoretical dynamic model of vehicle-trailer combination should be studied.

But there happened more or less traffic accidents when driven them. In order to improve the stability of central axle trailer, the theoretical dynamic model of vehicle-trailer combination was studied in the paper, and its system eigenvalues was analyzed.

2. VEHICLE SYSTEM MODELING

In order to evaluate the performance of vehicle, the model with 3 degrees of freedom (DOF), representing the vehicle combined with central axel trailer, should be briefly established.

2.1 THREE DOF BASELINE VEHICLE MODEL AND ANALYSIS

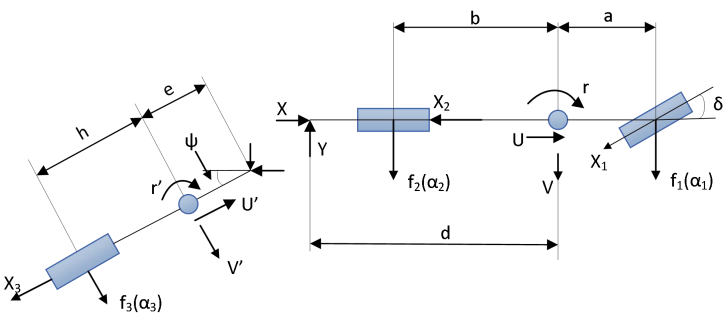


Fig.2: the Baseline Vehicle Model

The equations on the motions of the vehicle can be expressed as follows:

$$m_1(\dot{U}-Vr)=-X_1\cos\delta-X_2+X \quad (1)$$
$$m_1(\dot{V}+Ur)=f_1(\alpha_1)+f_2(\alpha_2)+X_1\sin\delta-Y \quad (2)$$
$$I_1\dot{r}=af_1(\alpha_1)-bf_2(\alpha_2)+aX_1\sin\delta+dY \quad (3)$$

The equations on the motions of central axle trailer are written as:

$$m_2(\dot{U}'-V'r')=-X_3-Y\sin\psi-X\cos\psi \quad (4)$$
$$m_2(\dot{V}'+U'r')=f_3(\alpha_3)+Y\cos\psi-X\sin\psi \quad (5)$$
$$I_2\dot{r}'=hf_3(\alpha_3)-e(-Y\cos\psi+X\sin\psi) \quad (6)$$

SURPOSE:

The towing and trailing units are connected at the articulation joint. The velocities and accelerations at the point expressed in the coordinate systems fixed with the tractor and trailer must be equal. Respectively this allows the trailer equations to be written in terms of the tractor fixed coordination system. Based on the following assumptions,the equations are linearized:

(1)The forward speed U is a constant and the forward motion equation (1) is ignored

(2)Small angle approximations are used:

$$\cos\psi=1,\ \sin\psi=\psi \quad (7)$$

(3)All products of variables are ignored.

(4)Linear tire model is used.

For zero initial conditions:

$$\dot{\psi}=r-r' \quad (8)$$

The linearized equations of motion can be written in the state-space form as

$$M\{\dot{x}\}+D\{x\}+F\delta=0 \quad (9)$$

where

$$\{x\}=\{V,r,\dot{\psi},\psi\} \quad (10)$$

The matrices M,D, and F, are respectively provided in the Appendix. And notation for vehicle unit is also in the Appendix .

Table1 - The baseline structural parameters

Description	Notation	Value
Tractor mass(Kg)	m_1	7850
Tractor yaw inertia(Kg·m^2)	I_1	50960
Tractor dimension(m)	a	2
Tractor dimension(m)	b	3.6
Tractor dimension(m)	d	5.25
Trailer mass(Kg)	m_2	5300
Trailer yaw inertia(Kg·m^2)	I_2	29767.9
Trailer dimension(m)	e	6.11
Trailer dimension(m)	h	0
Front tire cornering stiffness(N/rad)	c_1	-113450
Rear tire cornering stiffness(N/rad)	c_2	-113450
Trailer tire cornering stiffness(N/rad)	c_3	-113450

2.2 EIGENVALUE ANALYSIS

In order to evaluate the system stability of central axel trailer, the eigenvalues of the system can represented to investigate the effect of various parameters. The eigenvalues are functions of the vehicle forward speed U. If the system matrix has a pair of complex eigenvalue as

$$s_{1,2}=R_e\pm j\omega_d \quad (11)$$

The damping ratio, ζ ,can be determined by

$$\zeta = \frac{-R_e}{\sqrt{R_e^{\,2} + \omega_d^{\,2}}} \qquad (12)$$

(1) If ω_d=0, the eigenvalues are real numbers. When Re<0, the damping ratio is 1, ζ =1 ,the system is in monotonic convergence. When Re>0, the damping ratio is -1, ζ =-1, the system is in a monotonous divergence state and the vehicle is unstable. There may be folding or tailing.
(2) If $\omega_d \neq 0$, the eigenvalues are plurals.When Re<0, the damping ratio is between 0 and 1, 0< ζ <1,the system is in a state of oscillation convergence. When Re>0, the damping ratio is between -1 and 0, -1< ζ <0, The system is in a state of oscillation and divergence. The vehicle may be laterally vibrated and unstable.
Therefore, the dynamic response characteristics of the system can be analyzed according to eigenvalue which are judged by the damping ratio. Once the damping ratio is negative, it indicates an increasing amplitude of oscillation and instability.

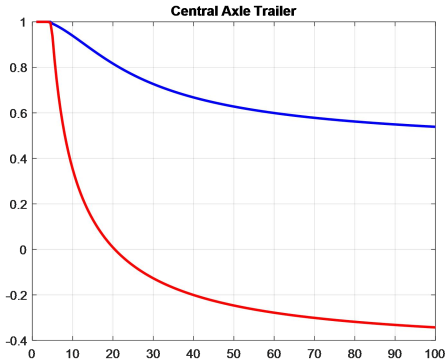


Fig.3: Damping Ratios Analysis

For the central axle trailer model, there are two pairs of complex eigenvalues with one damping ratio for each pair. Figure 3 shows the baseline vehicle model's damping ratios, varying as the functions of vehicle forward speed. Damping ratio 1 has the value close to 0.6 within the given speed range. Damping ratio 2 decreases with the increase of vehicle forward speed and it approaches 0 at the speed of about 20m/s. It is indicated that the critical speed is approximately 20 m/s, above which the vehicle system will be liable to an unstable motion mode.

3. DYNAMIC RESPONSES TO A STEERING INPUT

3.1 Test procedure

The structural parameters of central axle trailer are further compared through evaluating the corresponding vehicle dynamic responses to a simulated steering input. As shown in Figure 4, the steering input takes the form of one cycle of a sinusoidal wave, having a period of 3.14 seconds and a magnitude of 1 degree (0.0175 radians). By setting the initial values of all the state variables to 0, it is assumed that the car-trailer system is travelling along a straight line before the vehicle swerving. The maximum overshoot and the settling time are the primary performance measures used to evaluate the dynamic responses.

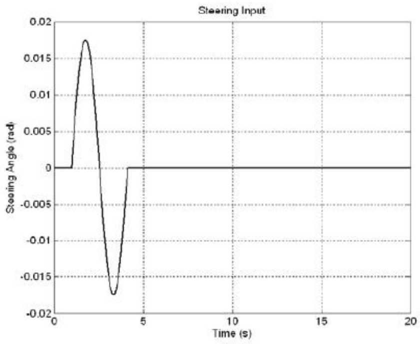


Fig.4: Simulated Steering Input

3.2 Dynamic responses

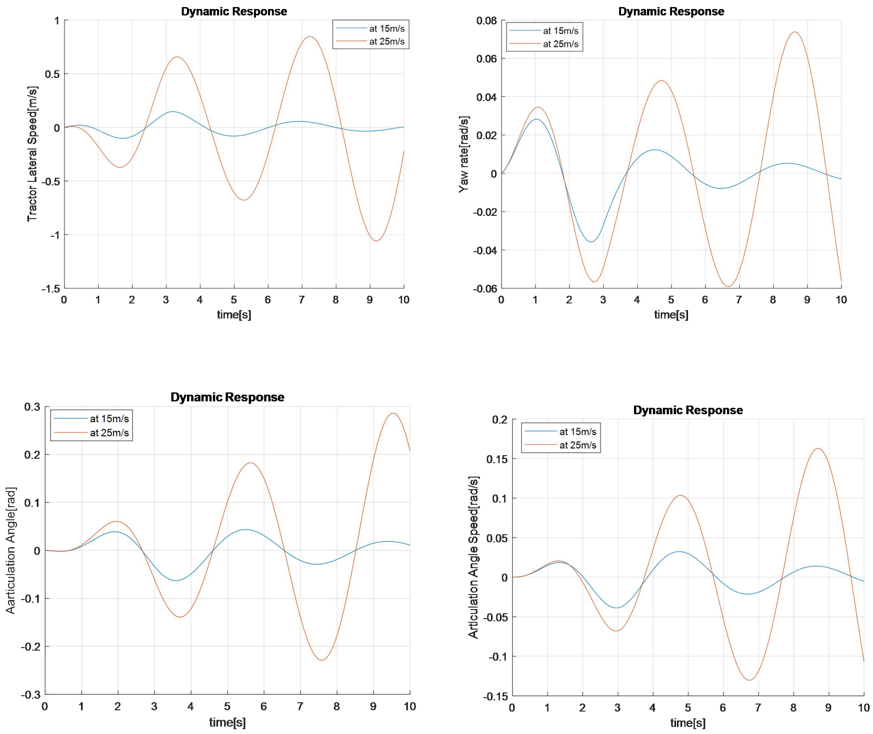


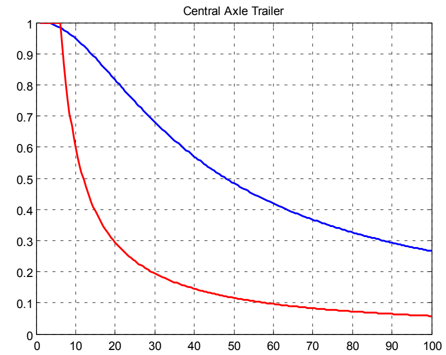
Fig.5: Dynamic Responses at Different Forward Speed

The above Figures 5 show the dynamics responses of baseline vehicle at different forward speed: 15 m/s and 25 m/s, respectively. For the baseline vehicle, the magnitude of the car's dynamic responses, such as tractor lateral speed, yaw rate, and articulation angle, increase with time at the speed of 25m/s and the vehicle will lose stability. In contrast, for the same vehicle, the magnitude of the car's dynamics responses decrease with time at the speed of 15 m/s and the vehicle is stable. The baseline vehicle's simulation results make sense since the vehicle's critical speed is 20 m/s, above which the vehicle will lose stability.

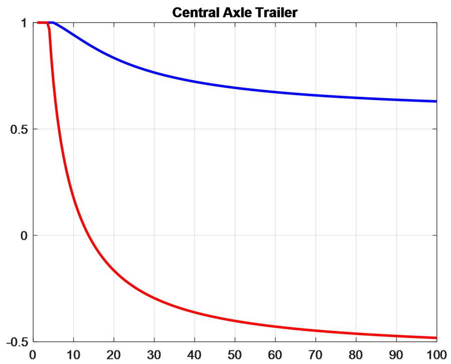
4. THE INFLUENCE OF DIFFERENT TRAILER PARAMETERS

After establishing the baseline trailer characteristics (Table 1), a further study on various trailer parameters was carried out. The aim of this study using the adjustable trailer was to alter one parameter at a time in order to assess the sensitivity of this complex dynamic system to single variables. In practical applications, in a real vehicle, it is likely that many important parameters are interrelated. However, the complexity of the combined vehicle–trailer system makes it difficult to establish the significance of individual factors if several are changed at once.

4.1 Trailer Mass Distribution



CG of trailer is 1m in the front of the axle position



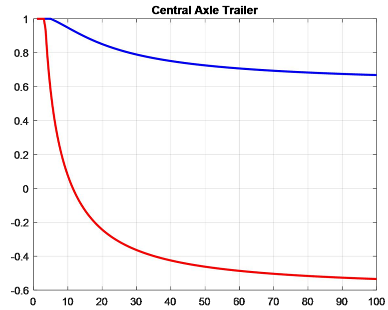
CG of trailer is 1m in the back of the axle position

Fig.6: CG of Trailer

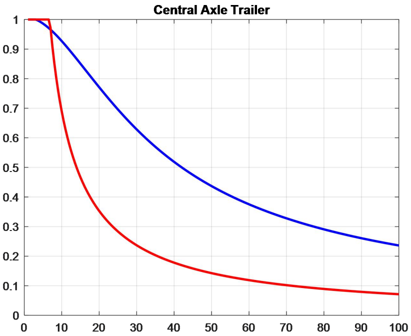
To study the effect of trailer loading on the system stability, the trailer was set to different masses distribution while maintaining other settings at the baseline level (Table 1). For example, an forward or backward movement in trailer mass distribution was accompanied by changing the CG position while other parameters remained unchanged. Figure 5 shows the damping ratio versus speed plots for various trailer masses. The trailer mass distribution had a significant effect on the high-speed stability. If the CG moved forward the trailer axle, the stability is increased obviously.

4.2 Trailer Axle Position

Analysis were performed with various axle position of trailer while other parameters remained the same as the baseline configuration. It was found that, the more the axle is located backward, the the more stable is the system, as shown in Fig.6. This can be attributed to the fact that, when the trailer axle move backward, the trailer lateral tyre forces act on a larger lever arm with respect to the tow hitch and this helps to stabilize the trailer oscillation.



the axle position move forward 1m, the CG is not changed

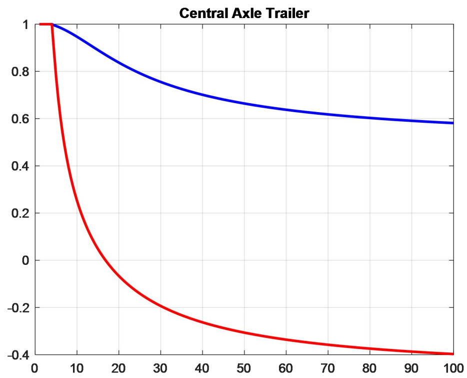


the axle position move backward 1m, the CG is not changed

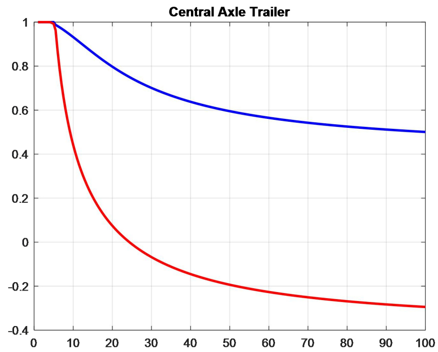
Fig.7: the Axle Position

4.3 Trailer Length

Since the trailer length will affect the vehicle dynamic response, tests were repeated at two different trailer length settings for the trailer. Figure 7 shows the corresponding damping ratio versus speed plots. In general, the longer the trailer, the stabler is it, although the change is not very significant.



Trailer length decrease 2m



Trailer Length increase 2m

Fig.8:Trailer Length

5. Effects of Parameters on CT Lateral stability

As experimental study has been conducted to examine the efforts of important trailer parameters on the dynamic stability of a CT combination (Darling et al.,2009).The testing procedure and data processing approach were based on the corresponding methods specified by ISO-9815. To carry out the test, one parameter was varied at a time.

Table2- Optimized geometric variables(permitted to vary by ±2% from their nominal values)

Variables	Nominal values	Upper bounds	Lower bounds	Optimized values	Nominal U_{1c} (ms^{-1})	Optimized U_{1c}(ms^{-1})	Increase of U_{1c}(%)
a(m)	2	2.04	1.96	2.04 ↑	20.451	20.456	0.024
b(m)	3.6	3.67	3.53	3.67↑	20.451	20.681	1.124
d(m)	5.25	5.36	5.15	5.15 ↓	20.451	20.672	1.081
e(m)	6.11	6.23	5.99	6.23 ↑	20.451	20.910	2.244
h(m)	0	0.2	-0.2	0.2 ↑	20.451	24.450	19.554

As shown in Table 2, with the given range of variation of the distance, optimizing h alone leads to a 19.55% increase in critical speed. Therefore, optimizing h alone leads to approximately 81.36% contribution to the critical speed increase, resulting from the optimization of the geometric variable set (a combination of variables a, b, d, e, and h).

6. Conclusion

Stability is the most important parts of the performance of the center axle trailer combination. In order to improve the stability of central axle trailers, eigenvalue of vehicle-trailer combination system was analyzed. The results showed that its critical speed were only 20 m/s for a given example of central axle trailer. It is unstable when driven on the express road because the forward velocity is usually above the critical speed. Through the influence of different trailer parameters on the system high-speed stability was examined, It found that the dominant structural parameters affected the stability, which were the trailer mass distribution, trailer axle positions, and trailer length. The mass distribution is the most important influence factor on the stability, while the trailer length has less significant effect. So the suggestion is the GC of load should be in the front of trailer axle to enhance the stability .

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APPENDIX

$$M = \begin{bmatrix} m_1 + m_2 & -m_2 d & -m_2 e & 0 \\ -m_2 d & I_1 + m_2 d^2 & m_2 ed & 0 \\ -m_2 e & m_2 ed & I_2 + m_2 e^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D = \frac{1}{u} \backslash \begin{bmatrix} -c_1 - c_2 - c_3 & -c_1 a + c_2 b + c_3 d + (m_1 + m_2) u^2 & c_3 (h+e) & -c_3 u \\ -c_1 a + c_2 b + c_3 d & -c_1 a^2 - c_2 b^2 - c_3 d^2 - m_2 du^2 & -c_3 d (h+e) & c_3 du \\ c_3 (h+e) & -c_3 d (h+e) - m_2 eu^2 & -c_3 (h+e)^2 & c_3 (h+e) u \\ 0 & -u & u & 0 \end{bmatrix}$$

$$F = \begin{bmatrix} c_1 \\ c_1 a \\ 0 \\ 0 \end{bmatrix}$$

Table 3 - Notation for vehicle units

Description	Notation	Value
Tractor mass(Kg)	m_1	7850
Tractor yaw inertia(Kg·m^2)	I_1	50960
Tractor dimension(m)	a	2
Tractor dimension(m)	b	3.6
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Trailer yaw inertia(Kg·m^2)	I_2	29767.9
Trailer dimension(m)	e	6.11
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Front tire cornering stiffness(N/rad)	c_1	-113450
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