THE ANALYSIS ON THE IMPACT COEFFICIENT OF BRIDGE WITH RESPONSE SPECTRUM

Zanxun Liu[1], Jujiu Liu[1] and Meixin Zhou[2]

[1]Testing Center of Municipal Engineering
 Guangzhou, China.
[2]Municipal Technology Institute, Guangzhou University
 Guangzhou, China.

ABSTRACT

We propose a new approach to calculate the impact coefficient by analyzing the relation between a vehicle's impact force on a bridge and the dynamic characteristics of a vehicle and a bridge, the plainness of bridge surface. The analysis is performed in the framework of computational structural mechanics and stochastic process theory. We also illustrate, with examples, how this new approach can be applied, with comparisons made between the new approach and two existing methodologies of calculating the impact coefficient: one that had been widely adopted by Chinese Design Code of Bridge and Culvert, and another that was used in the Canadian Quebec Design Code of Bridge and Culvert.

INTRODUCTION

With the rapid development of public transportation systems in China, the quality of highway construction has been undergoing constant improvement. New bridges are also adopting wider and lighter designs. At the same time, economic development throughout the country has been escalating the load of vehicles almost without a ceiling. As a result, one has to take into account as many aspects as possible of the impact of moving a load on a bridge, and in analysing internal forces of bridge structure, so as to eliminate large margins of errors. That is why a vehicle's impact force on a bridge has attracted more and more attention.

It is well known that the impact coefficient is related to many variables such as speed of a vehicle, dynamic characteristics of vehicle and bridge, the plainness of bridge surface, and so on. Currently, the impact coefficient is simply defined as a function of span in the Chinese Design Code of Bridge and Culvert. Especially, in the case of concrete bridges, the impact force is not considered when the span or influence line loading length exceeds 45m, which is definitely inaccurate at best. When the span exceeds 45m, the impact force is small, but its impact still exists。

To better determine the combinative impact of the above variables on the impact coefficient, in this paper we categorize the impact of dynamic characteristics of vehicle and bridge as dynamic multiply coefficient and categorize the impact of plainness of bridge surface, speed of vehicles on impact coefficient as road condition coefficient.

VEHICLE'S IMPACT FORCE ON BRIDGE

Basic assumptions
- For most of the cases, the first vibration mode has an overwhelming contribution to our analysis. Therefore, it is used to lay down the foundation for our computational model.
- It is assumed that vehicles always touch the bridge.
- Both bridge and vehicle have a sticky damping.

The computation of vehicle's impact on bridge
According to 达朗伯 principle, the dynamic equation of bridge structure reads：

$$M\ddot{Y} + C\dot{Y} + KY = P\left(\ddot{V} - \ddot{Y}\right)$$

which can be further written as：

$$\left(M + P\right)\ddot{Y} + C\dot{Y} + KY = P\ddot{V} \tag{1}$$

or equivalently,
$$\ddot{Y} + 2\xi\omega\dot{Y} + \omega^2 Y = \frac{P\ddot{V}}{M + P}$$

where：
$$\omega^2 = \frac{K}{M + P} \quad \xi = \frac{C}{2\sqrt{K(M + P)}}$$

Under the initial conditions that $Y(t=0) = 0$ and $\dot{Y}(t=0) = 0$, the integration of displace can be written as：

$$Y(t) = \frac{1}{(M + P)\omega_d}\int_0^t P\ddot{V}(\tau)e^{-\xi\omega(t-\tau)}\sin\omega_d(t-\tau)d\tau \tag{2}$$

For small damping, we have $C\dot{Y}$ 《 KY; when it is omitted, total inertia force of bridge F(t) can be written as：

$$F(t) = \left(M + P\right)\ddot{Y} - P\ddot{V} = -KY = -(M + P)\omega^2 Y(t) \tag{3}$$

Plugging (2) and (3), one has:

$$F(t) = -P\omega\int_0^t \ddot{V}(\tau)e^{-\xi\omega(t-\tau)}\sin\omega(t-\tau)d\tau \tag{4}$$

$$\text{Let} \quad S_{\max} = \omega\left|\int_0^t \ddot{V}(\tau)e^{-\xi\omega(t-\tau)}\sin\omega(t-\tau)d\tau\right|_{\max} \tag{5}$$

We then have
$$F_{\max} = P*S_{\max} \tag{6}$$

We can re-write (6) as：

$$F_{\max} = P*g\frac{\left|\ddot{V}\right|_{\max}}{g}*\frac{S_{\max}}{\left|\ddot{V}\right|_{\max}} = W*\frac{\left|\ddot{V}\right|_{\max}}{g}*\frac{S_{\max}}{\left|\ddot{V}\right|_{\max}} = W*K_V*\beta \tag{7}$$

where：W=Pg is the half weight of bridge-span.

$K_V = \dfrac{\left|\ddot{V}\right|_{\max}}{g}$ is the road condition coefficient, which is essentially the ration between vehicle's maximal vertical acceleration and acceleration of gravity.

$\beta = \dfrac{S_{\max}}{\left|\ddot{V}\right|_{\max}}$ —dynamic mulplicity coefficient. It is multiplicity of vehicle acceleration through bridge structure.

COMPUTATION OF ROAD CONDITION COEFFICIENT K_V

It is well known that road condition coefficient K_V is related to plainness of bridge surface, speed of vehicles, dynamic characteristics of vehicle and bridge. Next, we present the spectral analysis of K_V.

Power spectrum of plainness of road surface
A. unevenness function r(x): it is often defined as the road surface difference in elevation along road longitudinal section. Many statistical analyses have shown that unevenness of road surface is strongly correlated with power spectrum of road surface. Such analyses – both based on Chinese cases and those of other countries – show that random unevenness of road surface has a normal distribution. Thus, most power spectrum of road surface can be approximated by the following formula：

$$S_q(\Omega) = C_{sp} * \Omega^{-W} \tag{8}$$

where： W—frequce index number, which is a constant whitout dimension.

$$\Omega \text{—space frequence, m}^{-1}。$$

C_{sp} —unevenness coefficient of road surface, the dimension of which changes with W.

It was in 1972 that International Standard Assosiation adopted ISO SC2/WG4 recommended by British MIRA. This is a standard that categorizes highways based on their spectral analyses. This standard defined five categories of highways based on their different power spectra, and a mapping relation can be established between the highway rank and the corresponding C_{sp} and W.

Spatial power spectrum $S_q(\Omega)$ and temporal power spectrum

For a vehicle that travels at speed u on a road with a space frequence as Ω, the temporal frequencyfcan be written as：

$$f = u * \Omega \tag{9}$$

According to[6], the relation between spatial power spectrum $S_q(\Omega)$ andtemporal power spectrum $S_q(f)$ reads：

$$S_q(f) = C_{sp} * u * f^{-W} \tag{10}$$

The computation of $S_{\ddot{V}}(f)$ -the response power spectrum of vehicle acceleration due to unevenness of road surface:

The vibration differential equation of a sigle-freedom-degree vehicle can be written as：

$$P\ddot{V} + C_V \dot{V} + K_V V = C_V \dot{q} + K_V q \tag{11}$$

Where： P—mass of vehicle V—vehicle's displacement from the balance position

C_V—vehicle's damping K_V—vehicle's rigidity

q—unevenness of road surface

By the definition of frequence response function, integrating equation (11) givesthe acceleration$\ddot{V}$ as a function of the unevenness of road surface q：

$$H(j\omega)_{\ddot{V}-q} = \omega^2 * \frac{1 + j2\psi\lambda}{1 - \lambda^2 + j2\psi\lambda} \tag{12}$$

Where： $\lambda = \omega / \omega_0$—ratio of frequence（$\omega$ is frequence of unevenness of road surface, ω_0 is frequence of vehicle）

$$\psi = C_V / 2\sqrt{K_V p} \text{—relative damping coefficient}$$

Because road condition coefficient is solely determined by the amplitude of frequence response, we obtain：

$$| H(j\omega)_{\ddot{V}-q} | = \omega^2 * [\frac{1+4\psi^2\lambda^2}{(1-\lambda^2)^2+4\psi^2\lambda^2}]^{0.5} \tag{13}$$

Because the mean of $\ddot{V}$ is zero, its variance can be written as：

$$\sigma^2_{\ddot{V}-q} = 2\int_0^\infty |H(f)_{\ddot{V}-q}|^2 * S_q(f)df \tag{14}$$

Plugging equation (10) into the above formular, we have

$$\sigma^2_{\ddot{V}-q} = 2Csp * u * \int_0^\infty |H(f)_{\ddot{V}-q}|^2 * f^{-W}df \tag{15}$$

Because the output of any linear system has normal distribution if its input is normal, $\ddot{V}$ must demonstrate a normal distribution.We know that the probability of $|\ddot{V}| < 3\sigma_{\ddot{V}-q}$ is 99.73%; therefore, for any given road surface, we take $3\sigma_{\ddot{V}-q}$ as the maximum acceleration output for such a road. As a result, the road condition coefficient of such a road surface is：

$$K_V = \frac{|\ddot{V}|_{max}}{g} = \frac{3\sigma_{\ddot{V}-q}}{g} \tag{16}$$

The computation of dynamic multiplicity coefficient β

One way to obtain the value of Smax in β is through the calculation of the Duhumel Integral of equation (5).

However, because $\ddot{V}(t)$ here is the recorded vertical acceleration of the vehicle, its change in time cannot be written as a simple function.Thus, Smax cannot be obtained through Duhumel Integral.The only way that we can think of is through numerical integration of time-acceleration curve recorded with instrument.

For a record of an acceleration, if ξ is given, one can draw a $\beta-\omega$ curve.
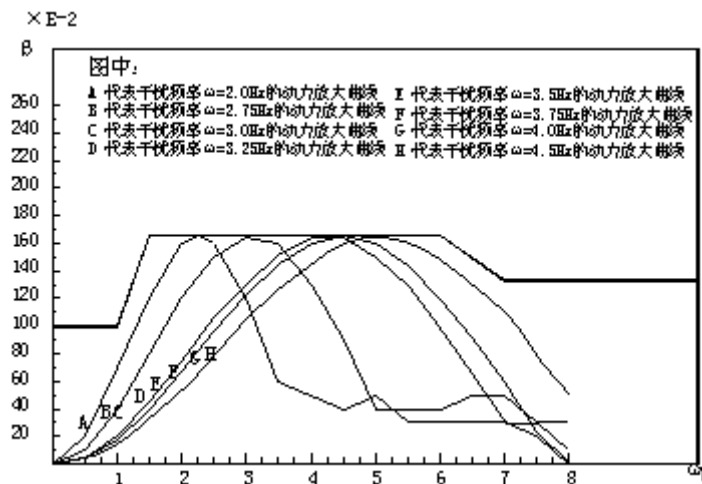

Figure 1. $\beta-\omega$ curve.

For a series of acceleration records, one can draw a series of $\beta-\omega$ curves. A β spectral curve can then be drawn by taking the average of these $\beta-\omega$ curves. For discussing the regularity of $\beta-\omega$ curves.

According to the basic features of Fourier transform, oscillation of any form can be represented as the linear combination of a series of harmonic oscillations of different frequencies, different amplitudes, and phases. For each harmonic oscillation, its frequency is a multiple of a basic frequency, and it corresponds to an eigenmode of the oscillation. To simplify, we write $\ddot{V}(t) = a\sin(\omega_0 t)$, i.e. we only keep the eigenmode of the oscillation, where: ω_0 is the frequency of the vehicle. In Figure 1, we give β spectrum curve for different ω_0. This figure shows that β arrives maximum when the frequency of vehicle is close to that of bridge; or else β is very small. Thus, it is impossible that all different harmonic components of one oscillation stay close to the resonant frequency of the structure. In fact, these components must stay away from one another. Thus the β curve in figure 1 can response approximately the true β curve.

EXAMPLES AND DISCUSSIONS

In the following example, we take a simple beam bridge with span L=19.56m, $\omega = 2.0Hz$ and an arch bridge without hinge with L=19.5, $\omega = 3.9Hz$.

By Chinese requirements: simple beam bridge $\mu = 0.19$, arch bridge $\mu = 0.20$

By Canadian (Quebec) requirements: simple beam bridge $\mu = 0.33$, arch bridge $\mu = 0.40$

Following the approach discussed above in this paper, we have: $\beta =1.65$

The first rank road surface: Kv=0.136 $\mu = 0.23$; the second rank road surface: Kv=0.223 $\mu = 0.37$

To further illustrate how different parameters affect impact coefficient, we take the following table of different combinations of parameters:

Table 1. Different combinations of parameters.

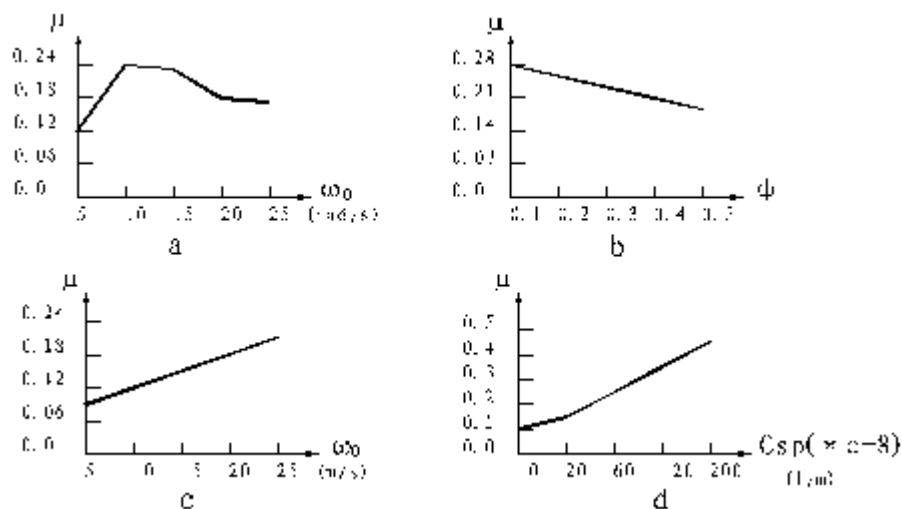| Parameters Combination No.	ω_0 rad/s	ψ	u m/s	Csp $\times 10^{-7} m^{-1}$
1	change	0.3	20	3.2
2	15	change	20	3.2
3	15	0.3	change	3.2
4	15	0.3	20	change

Figure 2. Impact coefficient and relative parameters.

Figure 2-a,b,c,d are computation result with different combinations of parameters in the above table so we can draw the following conclusions:

- There is no simple linear relation between impact coefficient and the inherent frequence of the vehicle, i.e., there is no increase-increase relation between the two quantities. It is only when frequency is within10~15rad/s (or 1.6~2.4Hz) that μ will take its amplitude, which means that most of the frequencies of the spectrum of high tape highway stay within this region; as a result, resonance of the vehicle appears.

- When damping of vehicle enhances, impact coefficient reduces, which means that damping of vehicle has important control of its oscillation, and an increase of damping of vehicle will reduce vibration of vehicle.

- The higher the speed of the vehicle, the larger the impact coefficient. This is mainly due to the fact that contributions of different frequencies of the road surface power spectrum (in time, which is due to acceleration or deceleration of the vehicle) change. In other words, when acceleration happens, high frequencies of the road surface power spectrum make more contributions, which causes severer oscillation of the vehicle.

- Changes of the impact coefficient is mainly due to the changes of the evenness of road surface. That is, the more even the road surface is, the smaller the impact coefficient will become; on the contrary, the worse the road surface condition is, the large the impact coefficient will become. Note that such relation is not linear: a worsening road surface condition can exponentiate impact coefficient

- According the approach to calculating impact coefficient that is presented in this paper, for the first rank highway, the impact coefficient that we obtained are smaller than those of Canada, but greater than those of China. For the second rank highway, the impact coefficient are greater than those of Canada for simple beam bridge, but smaller than those of China for arch bridge. Therefore, requirements set by the Chinese standards cannot guarantee safety.

- In summary, because the Chinese requirements only consider the relation between impact coefficient and span, our new approach to calculating impact coefficient is superior to the existing ways of such calculations and deserves more attention. As for the Canadian (Quebec) requirements, even though it takes into account the relation between impact coefficient and the dynamic characteristics of the structure, it nevertheless omits the unevenness of the road surface and its impact on impact coefficient. Based on such consideration, we believe that our approach that is presented in this paper can serve better conditions in practice.

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