Solid axle dynamics

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On a half car representation the dynamic response of a three degrees of freedom linear model is studied using lateral stiffness between the wheel and the road surface. Road vertical input in spectral and impulse form and lateral input in form of impulse of the centrifugal force are used to get five output performance criteria in form of frequency characteristic or impulse-effective values.

1 INTRODUCTION

The solid axle is an old and wide used technology in vehicles. It has several practical advantages which are well known. The most important performance feature is that the high instantaneous roll centre makes for low banking when cornering. Then there is the rotary movement of the axle which influences the ride performance.

The solid axle dynamics were given e.g. in [1]. The present paper takes into account some more aspects (seat, weight influence, lateral input) and brings the output criteria in impulse-effective form, too.

In this paper the model used is described by symbolic method, i.e. complex stiffnesses and complex amplitudes of time-depending values are used to compile the equations of motion. In this way the description is done with a lower number of equations and the stiffnesses, especially the lateral stiffness, can be handeled in a simple and objective form (the lateral stiffness can be shown in a schema).

The method deals only with harmonic vibrations. The impulse input is used in its spectral form.

2 MODEL OF THE VEHICLE

In this paper, only roll model is dealt with. (The bounce model is not influenced by the solid axle.) The model is shown in Fig.1. The vehicle body is denoted by its mass $2m_b$ and its gyration radius r_b. The height of the mass centre of the body is z_{tb}. The vertical displacement of the body at the radius $y_w/2$ is denoted by Z_b.

Between the body and the axle there is the suspension denoted by its stiffness k_b. It consists of a spring with spring rate k_{bre} and of a parallel damper with damping c_b. It is

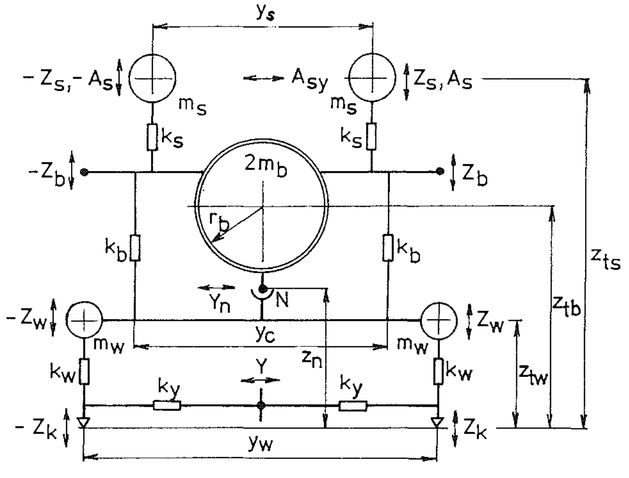

Fig.1 Model of the vehicle suspension

$$k_b = k_{bre} + i2\pi f c_b$$

The wheels are denoted by their masses m_w. For reasons of easy comparision with a model having an independent suspension, it is presumed that the mass of the solid axle is concentrated in the wheels. The radial stiffnesses k_w is

$$k_w = k_{wre} + i2\pi f c_w$$

where k_{wre} is the spring rate and c_w is the damping rate.

Z_w is the vertical displacement of the axle at the radius $y_w/2$.

Then there are the tyre lateral stiffnesses k_y which are composed according to Fig.2. of the real lateral tyre stiffness (spring rate) k_{wyre} with parallel tyre damping c_{wy} in series with the damping c_{pk}, which represents the slip of the tyre. So it is

$$\frac{1}{k_y} = \frac{1}{k_{wy}} + \frac{1}{k_{ypk}}$$

where

$$k_{wy} = k_{wyre} + i2\pi f c_{wy}, \quad k_{ypk} = i2\pi c_{pk}$$

Approximately it will be used

$$c_{pk} = a_g \frac{m_b + m_s + m_w}{v_x}$$

where v_x is the travel speed and a_g is the acceleration of gravity.

The track is y_w.

The distance between the seats is y_s, and the stiffness of the row of seats is

$$2k_s = 2k_{sre} + i4\pi f c_s$$

where k_{sre} is the spring rate and c_s the damping rate. The bodies of the seats are denoted by m_s. Z_s is the vertical displacement of the seat and z_{ts} is the height of the mass centre of the seat body.

The height of the joint is denoted by z_n, the lateral displacement of the joint by Y_n.

The vertical road input (unevennesses) are denoted by Z_k. The lateral input (the steering displacement when cornering) is denoted by Y and it affects the model by means of the lateral stiffnesses k_y.

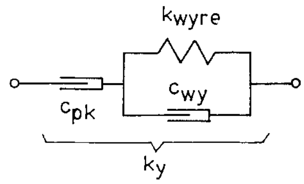

Fig.2 Schema of the lateral stiffness

The model is valid only for small angular displacements of the model. With the lateral input, usually bigger angular displacements occur so that the results are less precise.

3 EQUATIONS OF MOTION

According to Fig.1. the motion equations can be written:
The equation of moments to the axle about the joint N:

$$y_w k_w \frac{Z_k - Z_w}{2} = y_c^2 k_b \frac{Z_w - Z_b}{2y_w} -$$

$$2\eta_{SA}(z_{tw} - z_n) a_g m_w \frac{Z_w}{y_w} +$$

$$2(i2\pi f)^2 \left[\frac{y_w^2}{4} + \eta_{SA}(z_{tw} - z_n)^2 \right] m_w \frac{Z_w}{y_w} +$$

$$\eta_{SA}(i2\pi f)^2 (z_{tw} - z_n) m_w Y_n + z_n k_y (Y + 2z_n \frac{Z_w}{y_w} - Y_n)$$

The equations can be used for an independent suspension as well. In this case it is $\eta_{SA} = 0$, $z_n = 0, y_c = y_w$. For the solid axle it is $\eta_{SA} = 1$.

The equation of moments to the body about the joint N:

$$y_c^2 k_b \frac{Z_w - Z_b}{y_w} = 4(i2\pi f)^2 ([(z_{tb} - z_n)^2 + r_b^2] m_b +$$

$$(z_{ts} - z_n)^2 m_s + (1 - \eta_{SA}) z_{tw} m_w) \frac{Z_b}{y_w} - 4[(z_{tb} - z_n) m_b +$$

$$(z_{ts} - z_n) m_s + (1 - \eta_{SA}) z_{tw} m_w] a_g \frac{Z_b}{y_w} +$$

$$2(i2\pi f)^2 [(z_{tb} - z_n) m_b + (z_{ts} - z_n) m_s + (1 - \eta_{SA}) z_{tw} m_w] Y_n +$$

$$y_s k_s (y_s \frac{Z_b}{y_w} - Z_s)$$

In this equation, the difference from the independent suspension can also be seen. With the model of independent suspension the wheels roll with the body together.

The equation of vertical forces to the seat:

$$k_s (y_s \frac{Z_b}{y_w} - Zs) = (i2\pi f)^2 m_s Z_s$$

The equation of lateral forces:

$$k_y (Y + 2z_n \frac{Z_w}{y_w} - Y_n) = 2(i2\pi f)^2 [(z_{tb} - z_n) m_b Z_b +$$

$$(z_{ts} - z_n) m_s Z_b + (z_{tw} - z_n) m_w (\eta_{SA} Z_w +$$

$$(1 - \eta_{SA}) Z_b)] / y_w + (i2\pi f)^2 (m_b + m_s + m_w) Y_n$$

4 PARAMETERS, INPUT-OUTPUT SPECIFICATIONS

An example of the model behaviour will be given for the following vehicle parameters which are valid for a bus approximately:

$$m_s/m_b = 0.3, \quad m_s/m_w = 0.12, \quad r_b = 0.6\text{m}$$
$$z_{tb} = 0.9\text{m}, \quad z_{ts} = 1.4\text{m}, \quad y_w = 1.8\text{m}, \quad y_s = 1.2\text{m}$$

The spring rates and the damping rates for the body are chosen so that the basic frequencies and relative damping rates of the system are:

Table 1. Effective values by statistical road input in the range 0.5Hz - 32Hz (60km/h)

	A_s	A_{sy}	$Z_w - Z_b$	S_w/s_{st}	S_{wy}/s_{st}
	m/s^2	m/s^2	mm	-	-
IS	0.099	0.25	0.98	0.036	0.0041
SA0	0.086	0.23	1.02	0.042	0.0042
SA1	0.070	0.24	0.85	0.032	0.0085
SA2	0.025	0.18	1.22	0.041	0.0124

$$f_b = \frac{\sqrt{k_b/m_b}}{2\pi} = 1.41\text{Hz}$$

$$\vartheta_b = c_b / 4\pi f_b m_b$$

The spring rate and the damping rate for the seat are

$$f_s = \frac{\sqrt{c_{sre}/m_s}}{2\pi} = 3\text{Hz}, \quad \vartheta_s = 0.282$$

and for the wheels are

$$f_w = \frac{\sqrt{k_{wre}/m_w}}{2\pi} = 10\text{Hz}$$

$$\vartheta_w = \vartheta_{wy} = c_w / 4\pi f_w m_w = 0.025, \quad c_{wy} = c_w k_{wyre}/k_{wre}$$

To the input forms:

A statistical form of road unevenness input will be used, namely the spectrum of the antiphased uneneses which can be derived from the measured power density [1] in this way:

$$Z_k = Z_{k0} \sqrt{\frac{v_x}{v_0} \cdot \frac{f_0}{f}} \cdot \sqrt{\frac{1}{2} - \frac{\gamma}{2}}$$

where the constant $Z_{k0} = 0.64$ mm is valid for a medium roughness surface, $v_0 = 1$ m/s, $f_0 = 1$ Hz. The coherence constant γ depends both on the track $y_w = 1.8$m and the ride velocity v_x so that

$$\gamma = \frac{1}{1 + (2\pi f y_w / 4.5 v_x)^4}$$

The fourth power in this formula, instead of the second power in [1], was introduced by present author to meet the assumption that the roll unevennesses are null with f=0. Then there is a correction coefficient

$$\frac{1}{1 + (r_w f / v_x)^2}$$

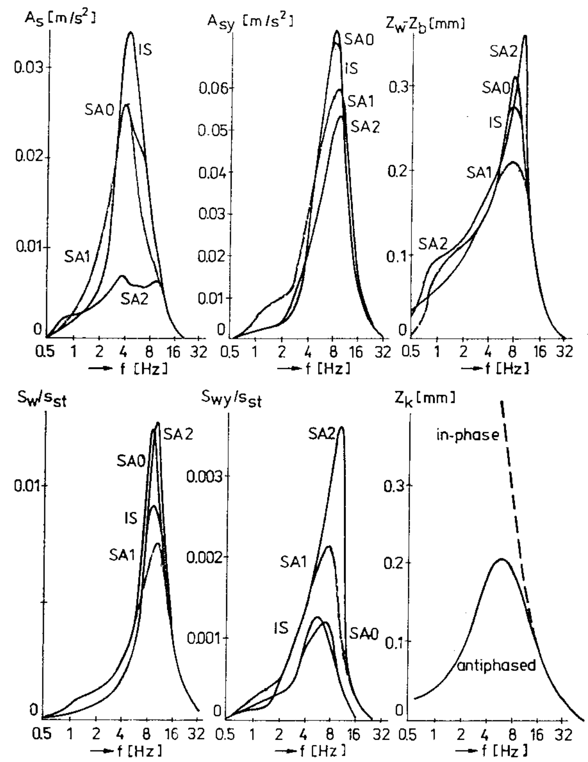

Fig.3 Amplitude frequency characteristics of the output criteria by statistical road input. Abbreviations:
IS independent suspension
SA0 solid axle with $z_n = 0, y_c = y_w$
SA1 solid axle with $z_n = 0.6$m$, y_c = y_w$
SA2 solid axle with $z_n = 0.6$m$, y_c = 1.2m$

multiplied to Z_k, where $r_w = 0.5$m is the wheel radius. This correction is also based only on an estimate.

With the road input, there is also a sinus impulse used. Its pick value $z_{ki} = 0.01$m is corrected according to its length x_i and the track y_w by multiplying with the value $2x_i/y_w$ when $2x_i < y_w$ with $y_w/2x_i$ when $2x_i > y_w$. Values of z_{ki} are shown in Fig.4, too. The spectrum of this impulse, used for computation, is

$$\frac{\sin(2\pi f t_i)}{2\pi f} + \frac{t_i}{2}\left[\frac{\sin(\pi - 2\pi f t_i)}{\pi - 2\pi f t_i} + \frac{\sin(\pi + 2\pi f t_i)}{\pi + 2\pi f t_i}\right]$$

For the lateral acceleration, a trapezoidal impulse was used according Fig.5. Its spectrum is

$$\frac{1}{\pi^2 f^2}\cdot\frac{1}{1.2t_i + 0.4 - (0.8t_i - 0.4)}\cdot$$

$$[\cos(\pi f(0.8t_i - 0.4)) - \cos(\pi f(1.2t_i + 0.4))]$$

Following criteria will be used:
The vertical acceleration of the seat

$$A_s = (i2\pi f)^2 Z_s,$$

the lateral acceleration in the seat

$$A_{sy} = 2[(i2\pi f)^2(z_{ts} - z_n) - a_g]Z_b/y_w + (i2\pi f)^2 Y_n,$$

the coefficient of the dynamic forces

$$S_w/s_{st} = k_w(Z_k - Z_w/a_g(m_b + m_s + m_w),$$

and the coefficients of lateral dynamic forces

$$S_{wy}/s_{st} = k_y(Y - Y_n + 2z_n/y_w Z_w)/a_g(m_b + m_s + m_w).$$

Also the body-wheel displacement

$$Z_w - Z_b$$

is important.

With an impulse input, the output criteria will be given in impulse-effective values, i.e. in effective values of the whole answer related to the length of the input impulse. E.g. for seat acceleration A_s by road input it is

$$A_{sef}^2 = 2/x_i.\int_0^\infty A_s^2(x)dx$$

where A_s is the value from the amplitude frequency characteristic. (This is the definition of the impulse-effective value. Computations for this paper were made in the frequency range using the above mentioned spectrum of the impulse.)

5 THE COMPUTED RESULTS

In Fig.3 there are the amplitude frequency characteristics of the output criteria by statistical road unevennesses.

When compared with the independent suspension (IS), it can be seen that the solid axle suspension with $z_n = 0$ (SA0) invokes greater banking and higher vertical dynamic forces but less seat acceleration. The non-zero height of the joint (SA1) results in diminishing of the banking and of the seat accceleration. It also causes a great increase of the lateral forces.

When the basis of the suspension wheel-body is less then the track, i.e. when $y_c < y_w$, as it is in the praxis, then there is major improvement in vertical seat acceleration, but a deterioration in lateral forces (SA).

The effective values are shown in the Table 1.

As the antiphased statistical road input at low frequencies is much less than the in-phase statistical input (Fig.4, right down, the dashed line), the output criteria are also of little value. Big antiphased road input occurs with an impulse shaped big unevenness, which has to be crossed at low speed. In Fig.4 there are the impulse-effective values of the criteria by a sinus-shaped antiphased unevenness plotted against the half-height impulse length that the independent suspension and the solid axle with zero height of the

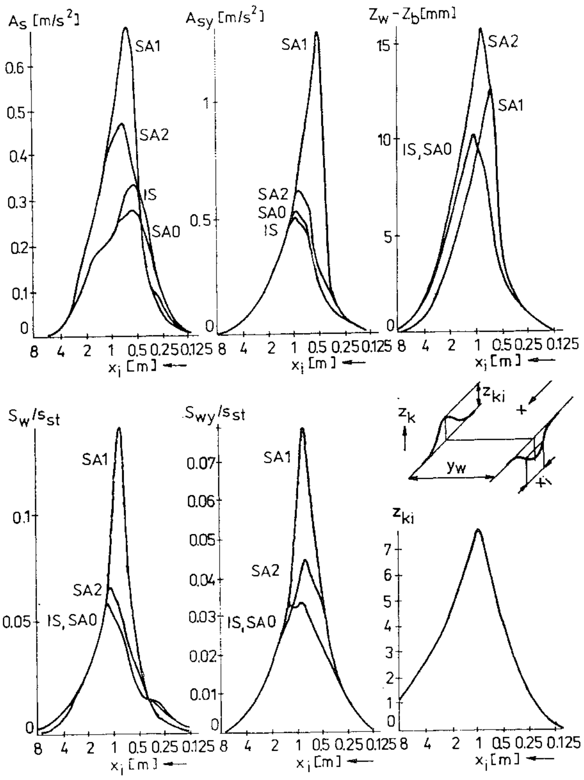

Fig.4 Impulse-effective values of the output criteria by a sinus-impulse unevenness. The input is shown at lower right-hand corner.

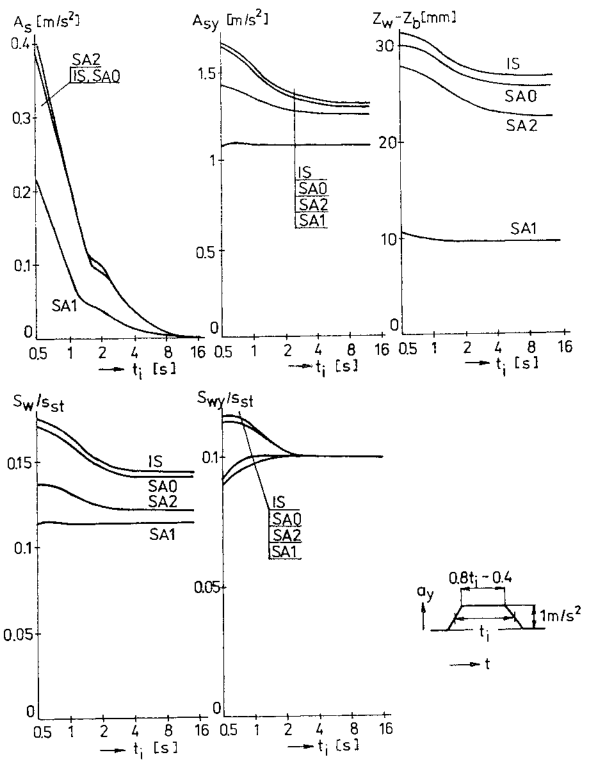

Fig.5 Impulse-effective values of the output criteria by an impulse of lateral input by cornering.

joint give very similar performance. The solid axle with $z_n = 0.6$m (SA1, SA2) gives a worse performance at all the used output criteria.

In Fig.5 there are the impulse-effective values of the output criteria by lateral input. (This input is produced by a lateral motion of the road surfice with an impulse acceleration according Fig.5 (right down). Fig.5 shows that suspensions IS, SA0 are near in their performance and that the non-zero height of the, joint improves every criterium - the variant SA1 more, the variant SA2 less. The most effected criterium is the displacement between wheel and body. The vertical forces coefficient is diminished by about 17%. (This result can be also useful for the power consumption of an active suspension [2].)

When the joint is placed above the mass center of the seats, i.e. when $z_n > z_{ts}$, then the body rolls in opposite direction in curves, so that the lateral acceleration in the seats is diminished. An exemple of this variant, used in some railway carriages, is not given in this paper.

6 CONCLUSIONS

In the sphere of the statistical input, the results do not unambiguously favor the solid axle nor the independent suspension.

An interesting result is that, with impulse road input at low travel speed, the solid axle is at a disadvantage.

With lateral input (by cornering), the solid axle has a clear advantage.

REFERENCES

1. MITSCHKE M. Dynamik der Kraftfahrzeuge, Band B: Schwingungen, Springer-Verlag Berlin 1984

2. ČECH I. A low-power active suspension and its bounce and cross model performance, I Mech E 1988 C422/88

LIST OF SYMBOLS

Constant physical quantities

a_g	acceleration of gravity, m/s^2
c_b	damping rate of the damper, Ns/m
c_{pk}	slip damping rate of the tyre, Ns/m
c_s	damping rate of the seat, Ns/m
c_w	damping rate of the tyre, Ns/m
c_{wy}	tyre damping rate in lateral direction , Ns/m
f	frequency, Hz
f_b	natural frequency of the body, Hz
f_s	natural frequency of the seat, Hz
f_w	natural frequency of the wheel, Hz
f_0	constant $f_0 = 1$ Hz
k_b	complex stiffness of the body-wheel suspension, N/m
k_s	complex stiffness of the seat, N/m
k_w	complex stiffness of the tyre, N/m
k_{wy}	complex stiffness of the tyre in lateral direction, N/m
k_y	complex lateral stiffness, N/m
k_{ypk}	complex slip stiffness, N/m
m_b	body mass, kg
m_s	mass of the body in the seat, kg
m_w	wheel mass, kg
r_b	gyration radius of the body, m
s_{st}	static load, N
t	time, s
t_i	impulse duration, s
v_x	travel speed, m/s
x	travel distance, m
x_i	half-height impulse length, m
y_c	distance between the springs, m
y_s	distance between the seats, m
y_w	track, m
z_{ki}	pick value of the sinus-impuls, m
z_n	height of the joint, m
z_{tb}	height of the mass centre of the body, m
z_{ts}	height of the mass centre of the seat, m
z_{tw}	height of the mass centre of the wheel, m

Complex amplitudes

A_c	lateral input acceleration, m/s^2
A_s	vertical acceleration in the seat, m/s^2
A_{sy}	lateral acceleration in the seat, m/s^2
S_w	wheel load, N
S_{wy}	lateral force, N
Y	lateral input displacement, m
Y_n	lateral displacement of the joint, m
Z_b	vertical body displacement at radius $y_w/2$, m
Z_k	road input, m
Z_s	vertical displacement of the seat, m
Z_w	displacement of the wheel centre, m

Quantities without physical dimension

ϑ_b	relative damping rate of the body
ϑ_s	relative damping rate of the seat
ϑ_w	relative damping rate of the wheel
ϑ_{wy}	relative damping rate of the wheel in lateral direction
i	imaginary unit $i=\sqrt{-1}$

Indizes

b	body
re	real component
s	seat
w	wheel (axle)

Note: Complex amplitude are denoted by capital letters, their instantaneous (time) values by the same lower-case letters. No attempt is made to distinguish constant complex variables from scalars or the complex amplitudes from their effective values.