SYNTHESIZING NONSTATIONARY, NON-GAUSSIAN WHEELED VEHICLE VIBRATIONS

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Abstract

This paper presents a novel technique by which non-Gaussian vibrations are synthesized by generating a sequence of random Gaussian processes of varying root-mean-square (rms) levels and durations. The technique makes use of previous research by the authors which shows that non-Gaussian vibrations from wheeled vehicles comprise of a sequence of Gaussian segments. Synthesis is achieved by creating a modulation function which is then applied, by means of a purposed-designed amplifier module, to a Gaussian random signal itself generated by a standard laboratory random vibration controller. This technique yields nonstationary, non-Gaussian random vibrations that statistically conform to the desired spectral and statistical (rms) distribution functions and are typical of those generated by wheeled vehicles. Results from experiments using such a Statistical Vibration Synthesizer system demonstrate the effectiveness of the method and its relevance for evaluating the effects of nonstationary random vibrations on consignments and passengers alike.

Keywords: vehicle vibrations, ride quality, synthesis, nonstationary, non-Gaussian.

1. Introduction

Although the vibrations generated by vehicles travelling over irregular terrain is a mature field of research, there does not exist a single definitive method to predict, analyze or synthesize the process (Sek, 1996). The current metric for both characterising and simulating transport vibrations, the average Power Spectral Density (PSD), is widely adopted and has been in place for some years. One major drawback of the PSD is that it effectively describes the average magnitude for each frequency band within the spectrum but does not contain information on time-variant parameters such as possible variations in amplitude or the time at which these variations occur. This is of no consequence if the process is both Gaussian and stationary. In such cases the nature of the signal is well defined by the normal distribution and its higher-order moments. However, as it has been repeatedly shown, road vehicle vibrations can often be significantly nonstationary and non-Gaussian mainly due to variations in pavement roughness and vehicle speed (Richards, 1990; Rouillard et al., 1996 & 2000; Rouillard, 2006). A typical example of this is shown in Figure 1.
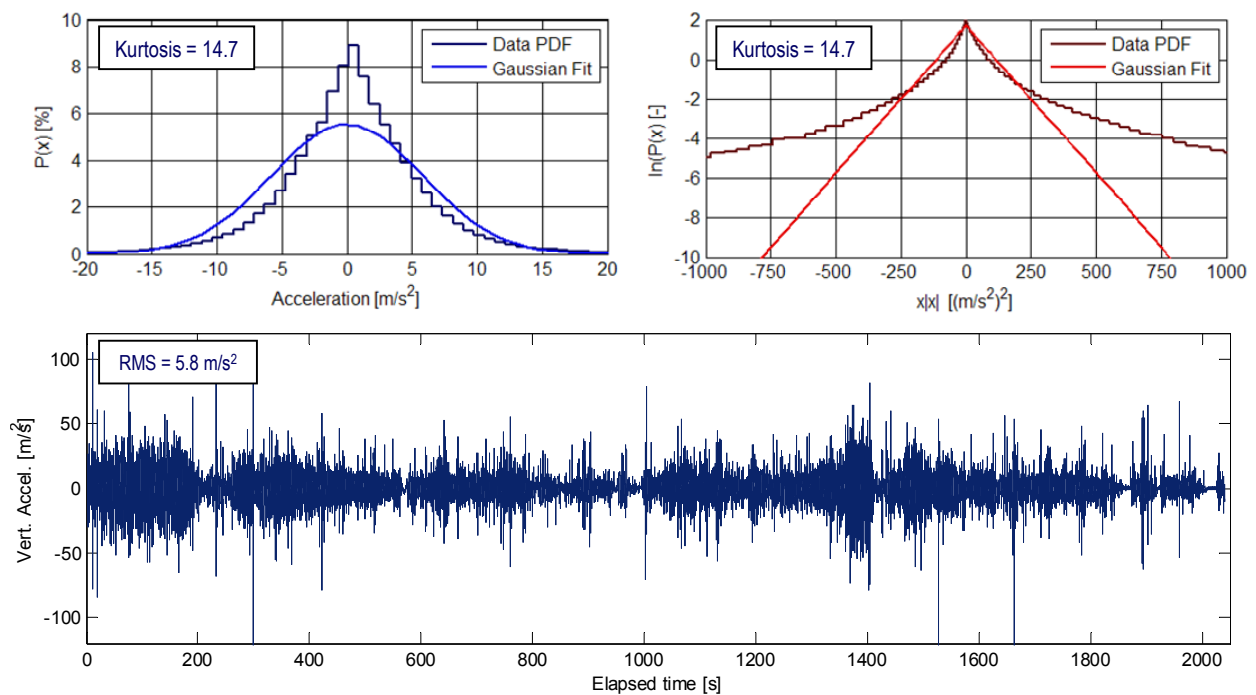

Figure 1: Example of the nonstationary, non-Gaussian nature of road vehicle vibrations. (Green line: best fitting Gaussian distribution).

The main consequence of ignoring the nonstationary and non-Gaussian nature of vehicle vibrations becomes critical when a single PSD function (usually the mean) of a vibration sample is used to synthesize these vibrations with laboratory vibration generators. In such cases, the resulting synthesized vibrations are unavoidably Gaussian and, consequently, fail to reproduce the random fluctuations in amplitudes that are inherent to the process.

In recent times, research aimed at characterising distribution vibrations as well as generating schemes for simulating these vibrations in controlled, laboratory environments have started to address issues relating to non-Gaussianity and nonstationarity. The research community is acknowledging that the characterisation of distribution vibration with the average Power Spectral Density (PSD) alone is not adequate. Despite this, the average PSD remains the

main (and often sole) ingredient used to synthesize random vibrations in most distribution laboratories around the globe. This paper describes the development and functionally of a practical technique for producing nonstationary, non-Gaussian vibrations in the laboratory.

2. Non-Gaussian vibrations

Many naturally-occurring and artificially produced random processes can be adequately characterised by the Gaussian distribution. Wheeled vehicle vibration is not one of them. Whenever random processes are studied, it is indeed very tempting to assume that the process is Gaussian as this enables the complete statistical characterisation of the process with the square root of the second-order statistic (the root-mean-square for zero-mean processes) and its frequency content by the average PSD. This is very significant relationship as it affords great simplicity for the analysis and synthesis of random vibrations. The importance of the Gaussian assumption is emphasized by W.M. Siebert: *The analyst would certainly be justified if he [sic] added to his familiar prayer, "Oh, Lord, please keep the world linear," the words "and Gaussian"* (Crandall, 1963).

One useful method for establishing how well a random process can be described by the Gaussian distribution - the Normality of the process - is by computing statistical parameters related to the higher order moments, m_n, of the process defined as:

$$m_n = \int_{-\infty}^{\infty} x^n p(x)\,dx = \frac{1}{N} \sum_{j=1}^{N} x_j^n \qquad (1)$$

The first two moments, respectively, produce the mean, μ_x, and the variance σ_x^2 while the third and fourth moments give the skewness, ν, and kurtosis, κ, parameters respectively. As vehicle vibrations are, in the main, symmetrical, only the kurtosis is useful for establishing non-Gaussianity:

$$\kappa = \int_{-\infty}^{\infty} \left(\frac{x - \mu_x}{\sigma_x} \right)^4 p(x)\,dx = \frac{1}{N} \sum_{j=1}^{N} \left[\frac{x_j - \mu_x}{\sigma_x} \right]^4 \qquad (2)$$

For a truly Gaussian process, the kurtosis, κ is three. Any deviation from these indicates that the process is non-Gaussian. There exists two main conditions for platykurtic ($\kappa < 3$) or leptokurtic ($\kappa > 3$) vibrations to occur:
   1. The random vibrations are stationary and inherently non-Gaussian,
   2. The random vibrations are composed of distinct segments each with an arbitrary duration and rms level, i.e. the vibrations are nonstationary. A special case of this condition exists when the segments are Gaussian. In the case of wheeled vehicles, this case may also be extended to include transient events generated by severe and sudden aberrations on the pavement surface (shocks).

It has been shown by Charles (1993) and Rouillard (2006) that, by virtue of varying vehicle speed and pavement roughness, wheeled vehicle vibrations are generally nonstationary. They showed that wheeled vehicle vibrations can be successfully modelled as a sequence of Gaussian segments of varying rms levels and durations (scenario 2). This approach is illustrated in Figure 2 which show how a number of Gaussian segments combine to produce a

non-Gaussian effect. Charles (1993) showed that the duration, D_i, and rms level, σ_i, of each segments are functions of the 'y-axis intercept', C_i, and the slope, m_i, of the distribution curve when the distribution is plotted on distorted scales.

$$\sigma_i = \sqrt{\frac{1}{2m_i}} \quad and \quad D_i = \sqrt{2\pi} \cdot \left(-\frac{1}{2m_i}\right) \cdot exp(C_i) \tag{3}$$
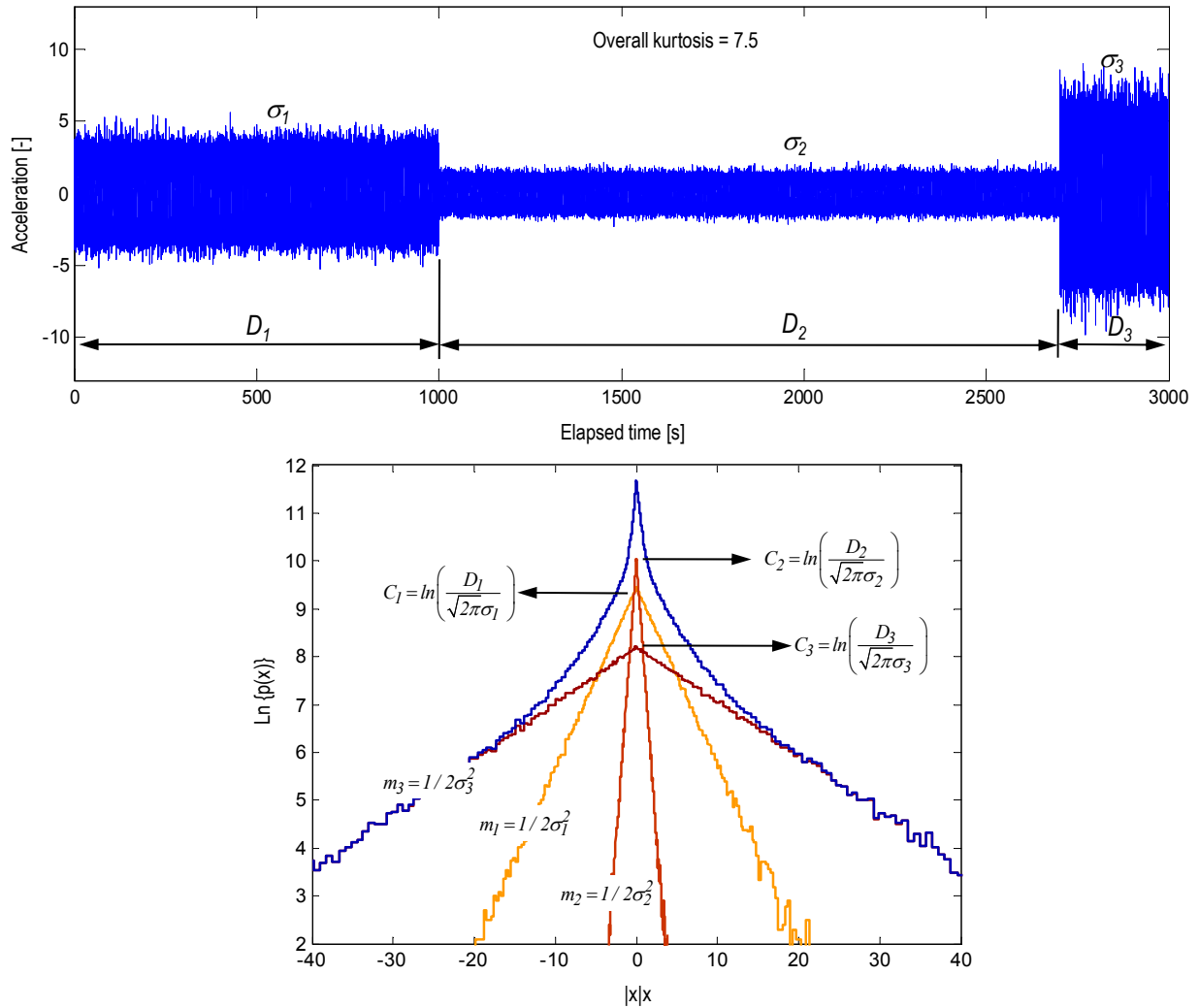

Figure 2. A random Gaussian sequence (top) and its corresponding Probability Density Function on distorted scales (bottom).

When the overall signal is modelled as a sequence of signals comprising i segments of varying durations and rms levels, the overall kurtosis, κ, can be expressed as a function of the vibration rms distribution $p_{rms,i}$ and the corresponding rms level, rms_i, as well as their individual kurtosis κ_i,

$$\kappa = \frac{\sum \kappa_i p_i rms_i^4}{\left(\sum p_i rms_i^2\right)^2} \tag{4}$$

Equation (4) indicates that $\kappa \neq 3$ is obtained when **a)** the segments themselves are non-Gaussian or **b)** when the vibrations are nonstationary – that is, the rms level of the segments vary. If in the second case, $\kappa_i = \kappa_{Gaussian} = 3$, the overall kurtosis depends only on the vibration rms distribution. In this case Equation 4 becomes:

$$\kappa = \frac{3 \sum p_i rms_i^4}{\left(\sum p_i rms_i^2 \right)^2} \tag{5}$$

When $\sum p_i rms_i^4 > \left(\sum p_i rms_i^2 \right)^2$ the vibration signal PDF is leptokurtic.

This can be represented by a statistical distribution of the rms level which, along with the overall kurtosis (computed from the rms distribution using Equation (5)), can be used to quantify the overall degree of non-Gaussianity of a process. Equation (5) is useful in that it enables the overall kurtosis of measured vibration data to be calculated directly from the rms distribution, which itself is readily available from field data recorders.

This approach, is well-suited to synthesizing nonstationary (hence non-Gaussian) vibrations. It requires that a Gaussian random signal, itself synthesized from a suitable spectral function, be modulated is such a way that produces the required stochastic variations in rms level. It is this modulation function that controls the level and nature of the nonstationarities and, as a result, the overall non-Gaussian character of the synthesized vibrations.

3. Gaussian Segment length distribution

Rouillard (2006) showed that, since the process can be modelled statistically as a sequence of Gaussian segments with varying standard deviations, there must exist identifiable boundaries (change-points) at which the transition from one segment to another occurs. The detection of such change-points should make it possible to determine the segment lengths as well as their statistical characteristics. A change-point detection algorithm, based on the cumulative sum (cum-sum) technique and enhanced by the inclusion of a bootstrapping algorithm was applied to numerous vertical vibration records from a variety of vehicles and routes (Rouillard, 2006). Vehicles used in the study included small utility trucks, vans, rigid trucks and semi-trailers with various suspension types and payloads; while routes included randomly-selected poorly maintained local roads, country roads, urban roads, and highways (motorways) located in Victoria, Australia. The broad variety of vehicles and pavement surface types selected for this study was aimed at minimising the influence of vehicle and pavement type on the results of the study which is itself aimed at road transport in general. Further details on the vibration records are given in (Rouillard, 2006). The results of the segment length distribution analysis yielded the following hyperbolic function (Rouillard, 2006):

$$p(s) = C / sinh(ks) \tag{6}$$

where s is the segment length and C and k are empirical constants. From a broad range of experimental data, C ≈ 4 and k ≈ ¼ for segment lengths measured in seconds

(Rouillard, 2006). The fact that this function can be easily integrated is advantageous as will be shown later.

4. Modulation Function

The modulation function is produced from the rms distribution and the segment length distribution. The rms distribution is normally established from field measurements and is usually dependent on the variations in pavement roughness and vehicle speed. The modulation vector needs to be created such that the random length of each stationary segment conforms to the statistical function for segment length distribution, Equation (6). This is achieved by firstly generating a sequence of uniformly-distributed random numbers scaled to the range of segment length. In order to transform a uniformly-distributed random variable, ξ, into any arbitrary non-uniform distribution, the cumulative distribution function is established and used as a transfer function applied to ξ. In the case of road vehicle vibrations, the cumulative distribution function is established by integrating the probability density function (Equation (6)) to give:

$$P(s) = \int C \, csc \, h(ks) \, ds = \frac{C}{k} \, ln \left\{ tanh \left(\frac{ks}{2} \right) \right\} \tag{7}$$

The random segment lengths synthesized in accordance with the hyperbolic statistical distribution model are combined with the desired rms distribution to produce the modulation vector. This is achieved by randomly allocating the RMS values from the rms distribution to a random segment length in the vector. This is repeated until every rms level represented in the distribution is fully allocated, which corresponds to the pre-determined duration of the synthesis. In order to ensure that each element of the rms distribution is filled at a uniform pace, the random allocation process was modified from a simple uniformly distributed random variable to a weighted allocation process that favours each rms bin in proportion to the distribution value. Figure 3 shows a typical random modulation signal along with its effect on a random signal.

An alternative method that produces similar results involves the generation of an appropriate number of required rms values that conform to the target distribution of the rms vibration. Since these rms values are discrete with a step equal to the rms bin width, uniformly distributed small fluctuations are introduced in order to simulate the continuous nature of the signal. These rms values are subjected to a uniformly random permutation and used to produce the modulation function that will be used to vary the gain applied to short segments of stationary, Gaussian signal generated by a RVC. Hence the synthesized signal attributes conform to the rms vibration distribution, the related kurtosis as well as the average PSD. The intensity of nonstationarity can be controlled by selecting different minimum segment duration.
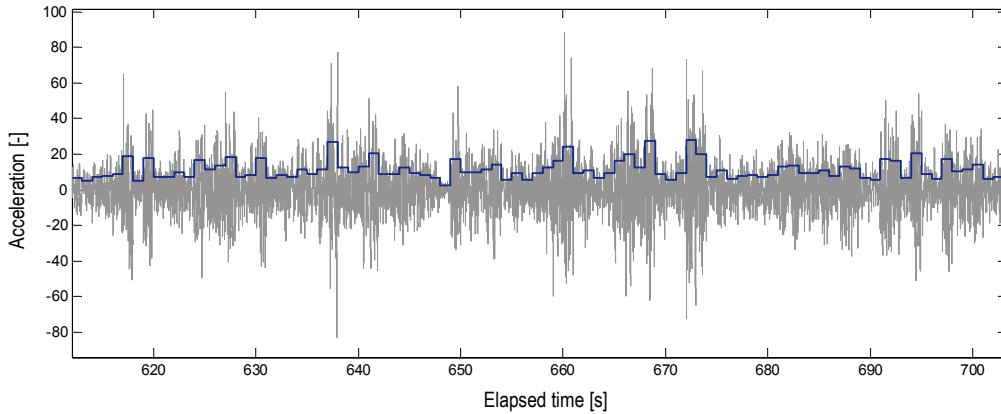
Figure 3: A random modulation function (dark line) along with modulated synthesized random vibrations (light line) for a typical case.

5.   Validation Experiments

The nonstationary random vibration synthesis method proposed above was experimentally validated using a typical laboratory-based random vibration simulation system - Random Vibration Controller (RVC) and vibration table - fitted with a specially-developed Statistical Vibration Synthesiser (SVS) module. The systems comprises specially-developed software to generate a modulation function from a statistical distribution of the rms level as well as a hardware module to amplify the command (drive) signal from the RVC. A schematic of the experimental arrangement is shown in Figure 4.
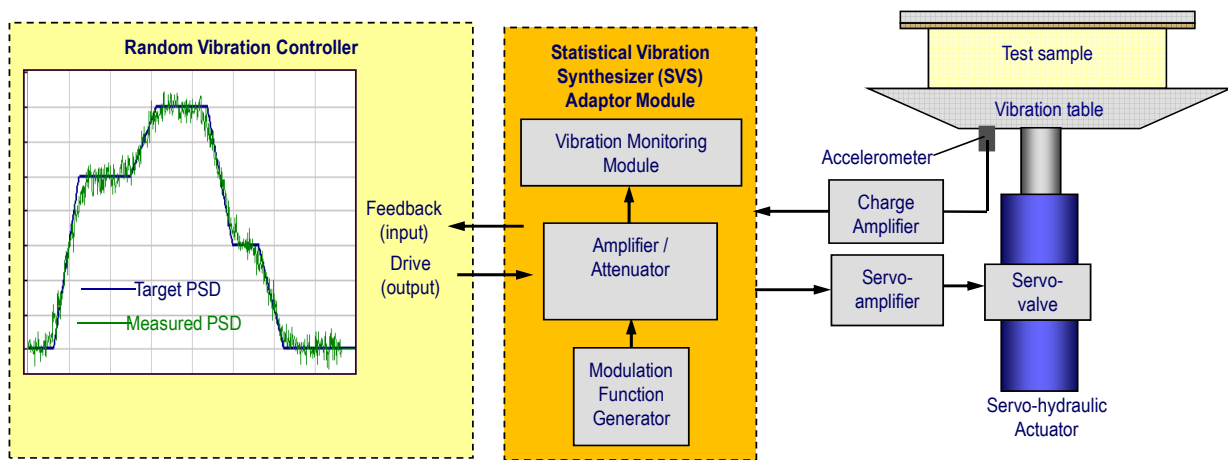

Figure 4: Experimental configuration for validation experiments.

The SVS system enables the user to configure various synthesis parameters. Importantly is the determination of the rms distribution which can be computed from field data or by specifying a schedule of road spectra and roughness (such as in ISO 8608), vehicle speed and vehicle type, namely, dynamic and suspension parameters (Alternatively, measured vehicle Frequency Response Functions can be used. Practical methods for the approximation of vehicle FRF from response data is the subject of current research by the authors).
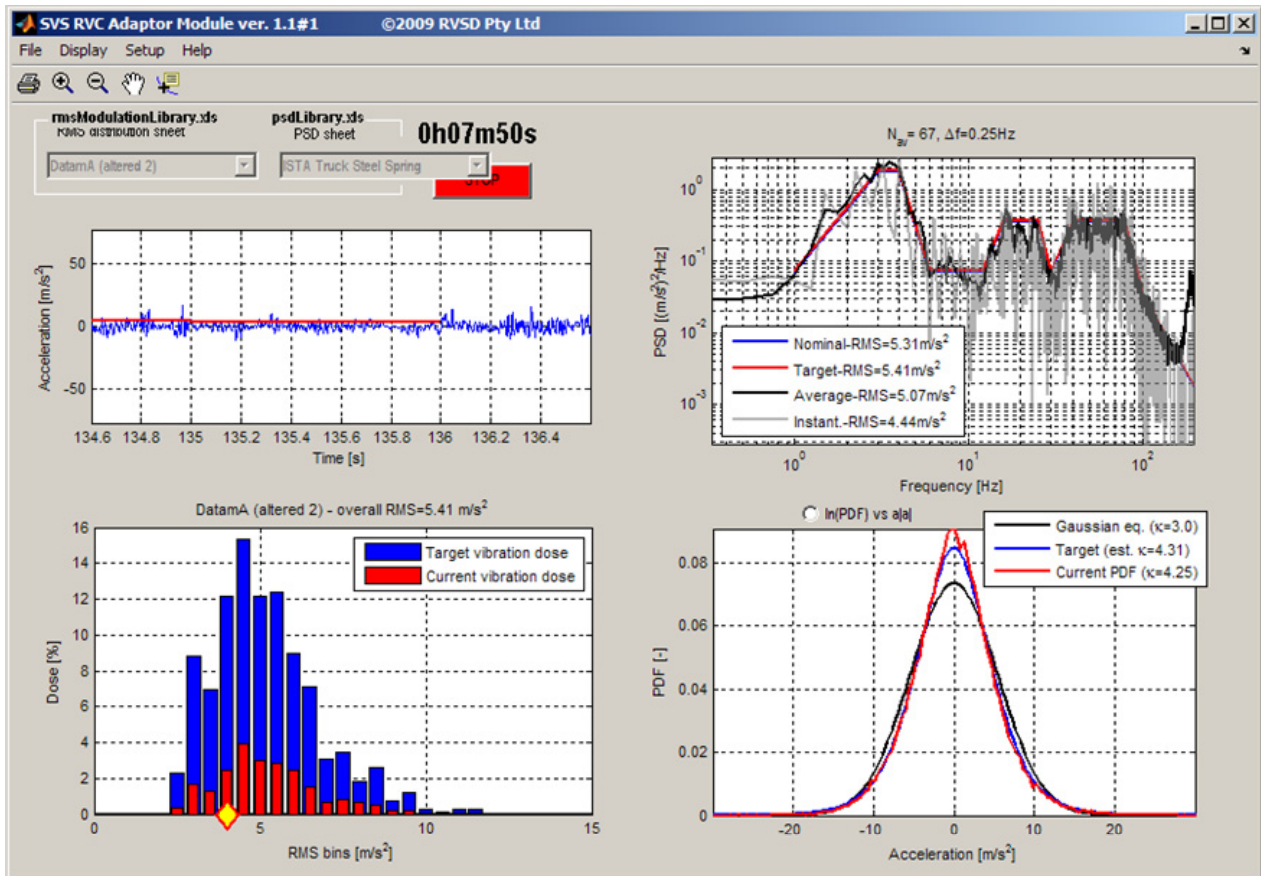
Figure 5: System monitoring view for the Statistical Vibration Synthesizer system during nonstationary vibration synthesis of a Truck with leaf springs (ISTA) with the rms vibration distribution defined by from measured data. Nominal rms level: 5.41 m/s², nominal kurtosis: 4.25, non-Gaussian signal range: ±46 m/s² (Gaussian: ±21 m/s²).

The Statistical Vibration Synthesiser System (SVS) includes provisions for continuously monitoring and display of the important vibration parameters during synthesis as shown in Figure 5 in which the system is shown with approximately ¼ of the time remaining.

Top right are shown the instantaneous (grey) and averaged (black) PSDs of the vibrations produced by the vibration table along with the target spectrum (red) and the ±3dB limits.

Bottom left are the target and (partially) realized rms vibration distributions.

Bottom right is the overall vibration PDF of the realized vibrations along with the best-fitting Gaussian distribution.

6. Results

Numerous validation experiments were carried out using statistical and spectral models calculated from a variety of configurations including measured data and standard PSD functions.   Figure 6 shows the signals for both the original vibration record and the synthesized (randomly modulated) vibrations for a typical, representative case.   The nonstationary character of the synthesized vibrations is clearly evident and, although quite different from the original signal (as expected), possess the same spectral and statistical characteristics as those of the original record used to generate the statistical models.   For comparative purposes, Figure 7 also shows stationary (unmodulated) random vibrations produced using a conventional RVC on its own.   Although the same overall rms level is reproduced, the absence of variations in rms level is clearly evident.
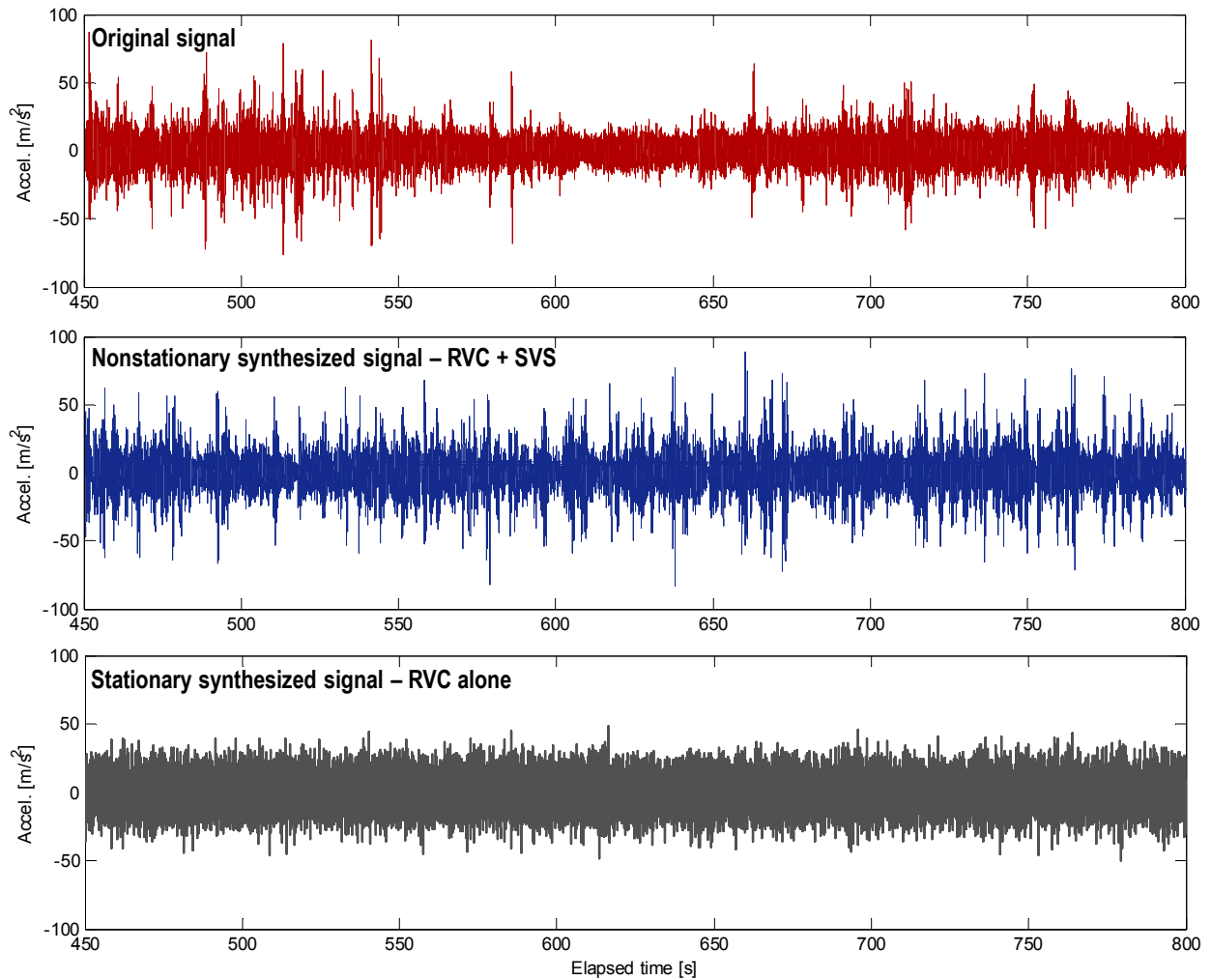
Figure 6: Original (top), synthesized nonstationary (modulated) vibrations (middle) and stationary (unmodulated) vibrations for a typical case.

Figure 7 shows the spectral and statistical character of vibrations synthesized by the SVS system for a variety of conditions. Results generated by the many validation experiments clearly demonstrate that the proposed method affords a practical solution for synthesizing nonstationary, non-Gaussian random vibrations that are realistic simulations of vibrations generated by transport vehicles.

Despite the general appearance of strong agreement, the kurtosis statistic of the original and synthesized vibrations do not always agree as shown in Table 1. This was further explored by computing the correlation between the original and synthesized distribution functions for the three representative cases shown. These are also included in Table 1.
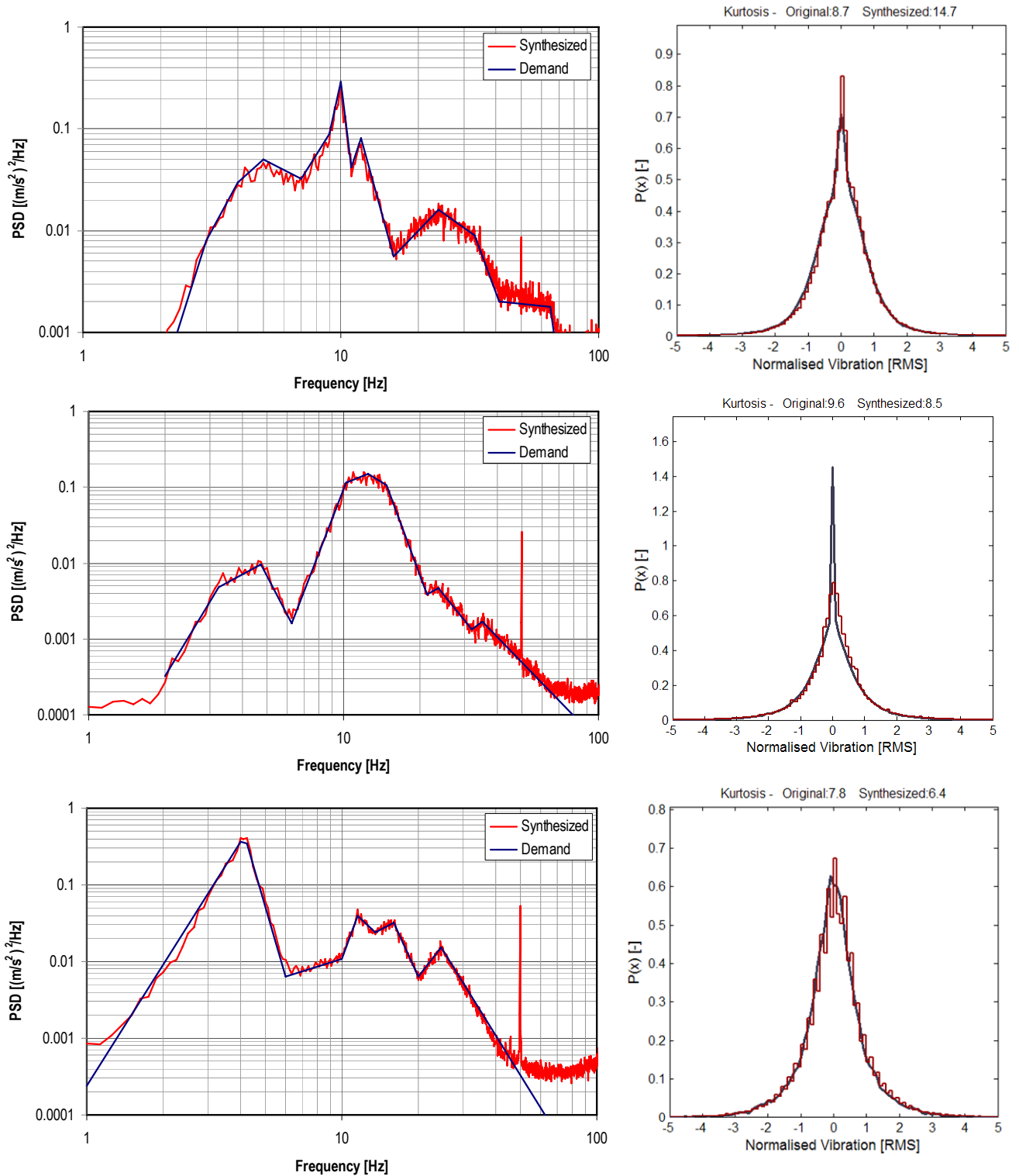
Figure 7: Demand and synthesized spectra (left) and PDF (right) of the original and synthesized vibrations for three typical cases:
(a): PSD and rms distribution: small utility vehicle on poor urban roads (Data MA).
(b): PSD and rms distribution: Prime mover + semi-trailer with Air-ride suspension loaded to 90% capacity travelling on country roads (Data MB).
(c): PSD and rms distribution: Numerical model of steel suspended truck travelling on a major country road (Data SA1100).

Table 1. Comparison of statistical parameters for original and synthesized vibrations.

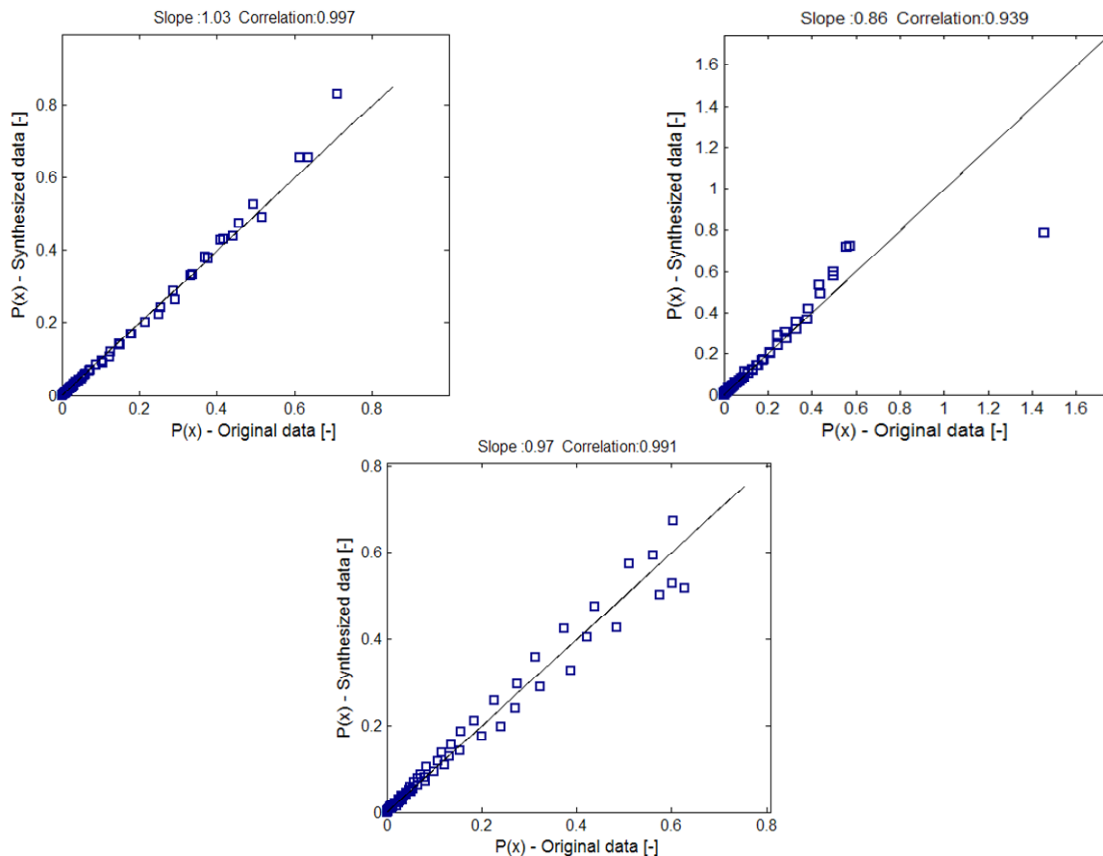| Statistical parameter		MA		MB		SA1100	
		Original	Synthesized	Original	Synthesized	Original	Synthesized
Overall RMS [m/s^2]:		12	12	2.3	2.3	0.82	0.82
Kurtosis :		8.7	14.7	9.6	8.5	7.8	6.4
PDF	Slope:	1.03		0.86		0.97	
	Corr. Coeff.:	0.997		0.939		0.991	
Inst. Mag. PDF	Slope:	1.01		0.91		0.97	
	Corr. Coeff.:	0.996		0.905		0.997	

Figure 8. Correlation plots of the synthesized and original PDFs for case MA (top left) , MB (top right) and SA1100 (bottom).

Scatter plots of the probability distribution estimates for all cases shown in Figure 8 reveal that the differences in the kurtosis statistic between the synthesized and original data are due mainly to outliers in the data. These are manifestations of the large differences in the PDF near the statistical mean of the process which, by virtue of approaching zero, are not significant to the overall process. This suggests the kurtosis statistic is not always an accurate indicator of the overall shape of the probability density functions on non-stationary processes. Generally, the agreement between the PDF of the synthesized and original vibrations are

confirmed by the reasonably good agreement values of the slope (where a slope of 1 is ideal) and correlation between the variables.

7.   Conclusions

The paper has described a novel system for synthesizing nonstationary, non-Gaussian random vibrations for the physical simulation of road vehicle vibrations. The method makes use of the fact that wheeled vehicle vibrations have been shown to be nonstationary and to consist of a sequence of Gaussian segments. This is usually manifested by an elevated overall kurtosis.

It was shown that the non-Gaussian nature of road vehicle vibrations is attributable to the inherent nonstationarity of the vibrations themselves caused primarily by variations in pavement roughness and vehicle speed. The paper proposes a model for computing the overall kurtosis for the rms distribution function and suggests that the technique is readily applicable for the synthesis on nonstationary, non-Gaussian vibrations by the use of a modulation function.

It has also been shown that the nonstationary (hence non-Gaussian) effect can be achieved by combining the rms distribution with a spectral function (PSD) of the vibrations. The paper describes how these stochastic fluctuations in rms level are produced in accordance with a statistical model describing the segment length distribution, itself obtained from vibration data recorded from a broad range of vehicle and route types. This new technique has been implemented into a computer-controlled system that is fully compatible with existing Random Vibration Controllers. When used in conjunction with existing laboratory vibration simulation equipment, the Statistical Vibration Synthesizer only requires that a statistical distribution of rms levels be available. The system incorporates facilities to generate the rms distribution by various means such as computing from field measurements or created by combining road roughness data, vehicle type and vehicle speedn.   Results show that vibrations synthesized with the new system were found to successfully emulate the stochastic variations in rms level that occur naturally during transport as well as to conform to the spectral and non-Gaussian statistical characteristics.

8.   References

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