

## Synthesising Spatially Repeatable Tyre Forces From Axle Load Probability Distributions

Will Goodrum graduated from the University of Virginia in 2007 with a B.S. in Mechanical Engineering, focusing on Materials Science. He is currently a PhD candidate at Cambridge University investigating Vehicle-Road Interaction.



W.J. GOODRUM, JR  
Cambridge University,  
Engineering Department  
United Kingdom



David Cebon is Professor of Mechanical Engineering at Cambridge University. He is a Fellow of the Royal Academy of Engineering, the Director of the Cambridge Vehicle Dynamics Consortium and the Managing Director of Granta Design Limited.

D. CEBON  
Cambridge University,  
Engineering Department  
United Kingdom

### Abstract

The work described in this report sought to characterise the spatial repeatability of a fleet of US 'Class 9' semi-trailers using axle load probability distributions as input. Pitch-plane vehicle models derived from 'per-vehicle' Weigh-In-Motion data were used to generate a 'reference' vehicle fleet for spatial repeatability calculations. Three strategies were investigated for simulating dynamic tyre forces with a similar level of spatial repeatability as the 'reference' fleet: randomised pitch-plane models, randomised quarter car models (both derived from axle load probability distributions), and phase-shifted quarter car models. Of these three strategies, the phase-shifting method reduced computation time by four orders of magnitude relative to the 'reference' case while approximating the 'reference' cumulative loading history with reasonable accuracy.

# 1. Introduction

## 1.1. Traffic Modelling for Pavement Design

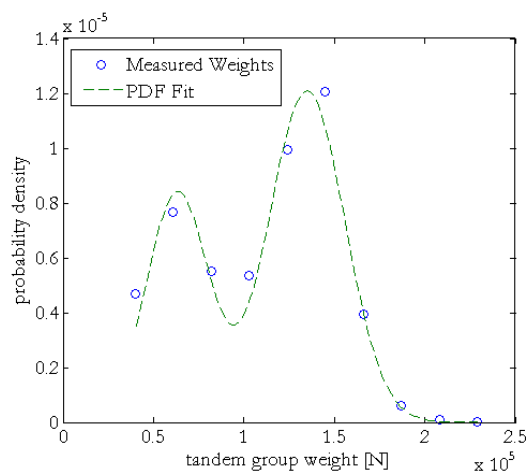
### *Approaches to Traffic Modelling*

Flexible pavements deform and fatigue under the repeated action of heavy vehicle traffic. Pavement design methods require accurate estimates of traffic loading. Traditionally, vehicle weight has been empirically related to decreased pavement serviceability through the Equivalent Single Axle Load (ESAL). This is calculated using the ‘fourth-power law’, as determined from the American Association of State Highway Officials (AASHO) Road Test (1958-1960) and codified in the AASHO Pavement Design Guide (Cebon 1999). ESALs implicitly incorporate a road damage relationship, which is independent of the structure of the road and mode of failure. Many researchers have, therefore, questioned their use (Gillespie et al. 1993; ARA 1999; Cebon 1999). In 1987, the US Long-Term Pavement Performance (LTPP) study began a large-scale field trial to investigate the effects of design and maintenance factors on pavement performance (LTPP 2006). High standard, quality-controlled traffic data has been available from LTPP Special Pavement Studies (SPS) sites since 2006 (LTPPINFO 2009). Data from all LTPP sites was used in the creation and validation of the American Association of State Highway and Transportation Officials (AASHTO) Mechanistic-Empirical Pavement Design Guide (ME-PDG) traffic module, where axle load probability distributions are used to quantify the traffic loading (ARA 1999).

### *Axle Load Probability Distributions*

Axle load probability distributions display the probability of the weights of a particular axle or axle group measured at a given site. In the ME-PDG, the pavement distress due to an axle group is calculated using probability distributions and the assumed number of vehicles. This more realistic characterisation of traffic than the traditional ESAL approach is a useful step forward for accurate pavement damage calculations (ARA 1999; Timm et al. 2005; Haider, Harichandran 2007).

Figure 1 is a tandem group load probability distribution calculated from ‘per-vehicle’ WIM data from the Florida LTPP SPS site (LTPPINFO 2009). Note that the probability density function (PDF) has two main peaks (or ‘modes’) corresponding to unladen vehicles and laden vehicles.



**Figure 1. Tandem axle group load probability distribution derived from WIM records of FHWA Class 9 vehicles taken at SPS site 12-0100 (Florida) (LTPPINFO 2009)**

Both ESALs and axle load probability distributions assume that the axle loads generated by heavy vehicles are static and therefore constant at all points along the road. In practice, heavy vehicles vibrate in response to rough road surfaces, generating dynamically varying tyre forces. These “dynamic tyre forces” or “dynamic axle loads” are known to be repeatable in space because heavy vehicles often travel at similar speeds with similar payloads, dimensions, suspensions, and tyres (Cole, Cebon 1992; Cole et al. 1996; Collop et al. 1996). Whole-life pavement response calculations account for repeatable loading by simulating the dynamic response of vehicles to a rough road surface (Collop, Cebon 1995). The challenge of whole-life modelling is to create the correct level of repeatability for the traffic fleet over the lifetime of the road (i.e. millions of vehicles), using a minimum amount of computation time. This paper describes an investigation of available methods for generating repeatable dynamic tyre forces from axle load probability distributions.

## **1.2. Objectives**

The objective of this study was to devise a calculation method that could model the spatial repeatability of a reference vehicle fleet as accurately and efficiently as possible.

## **2. Vehicle Modelling**

### **2.1. LTPP ‘Per-Vehicle’ WIM Records**

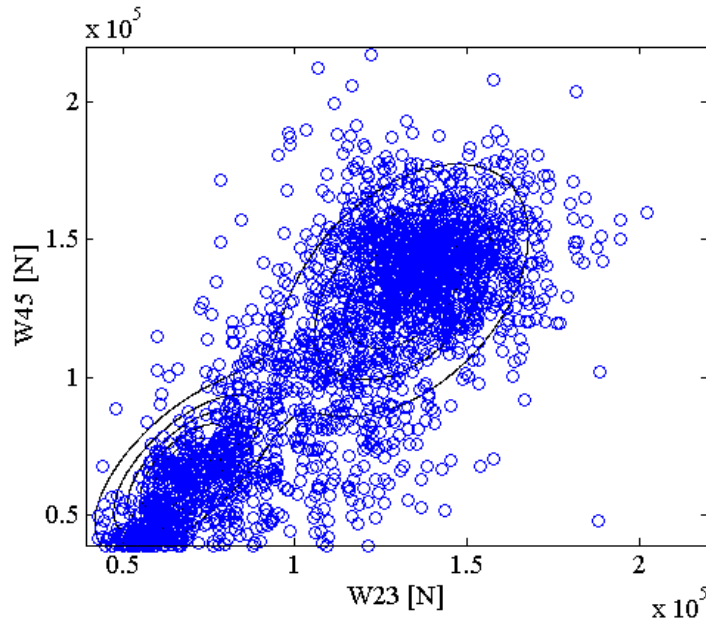
It was necessary to generate a ‘reference fleet’ of vehicles for comparison with the various traffic simulation methods examined in the study. For this purpose, vehicle models were derived from ‘per-vehicle’ WIM records of heavy goods vehicle traffic in 2006 on US Highway 27 between Belle Glade, Florida and Interstate-75 (Reel 2009). As a simplification, only FHWA ‘Class 9’ vehicles (three-axle tractor, two-axle semitrailer) were modelled.

The speed distribution for all fleets was assumed to be the same as for WIM data from the Texas SPS site 48-0100 (LTPPINFO 2009). A mean speed of 30.7 m/s (69 miles/hour) with a standard deviation of 1.7 m/s (3.7 miles/hour) was used. The relatively high mean speed was accepted for the Florida site since it was a similar rural highway with a speed limit of 29 m/s (65 miles/hour).

### **2.2. Multivariate Axle Load Probability Distributions**

It was found that the tractor tandem weights (W23) and trailer tandem weights (W45) of Class 9 vehicles were correlated. This is expected since heavily and evenly loaded trailers generate high loads on both tandem groups. Characterising this correlation requires a multivariate axle load probability distribution derived from the distribution shown in Figure 1. Figure 2 is a contour plot of the multivariate Gaussian mixture distribution fit to the tandem axle group weights from the Florida ‘per-vehicle’ WIM records, overlaid with a scatter plot of the same data. The fitted distribution adequately captures the spread of the data.

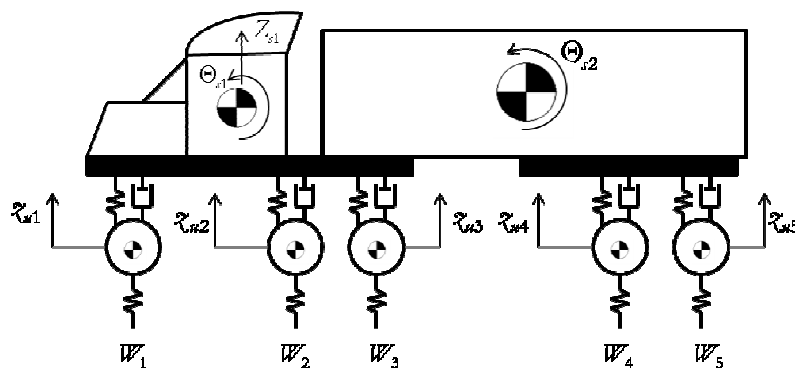
Correlation coefficients of 0.65 and 0.45 were fit to the unladen and laden peaks of the multivariate Gaussian distribution, respectively, using maximum likelihood estimation. It was also found that the steer axle group weight was uncorrelated with W23 or W45. Further details on the correlation of Class 9 axle groups can be found in (Goodrum, Cebon 2010).



**Figure 2. Contours of multivariate Gaussian mixture distribution fit overlaid with a scatter plot of trailer tandem weight (W45) versus tractor tandem weight (W23) (LTPPINFO 2009)**

### 2.3. Pitch-Plane Model

As stated in Section 2.1, only FHWA Class 9 vehicles were simulated for this study. Figure 3 is a schematic of the Class 9-derived pitch-plane model. Previous work by Cebon and Cole formed the basis for this model (Cebon 1985; Cole, Cebon 1991, 1992, 1995).

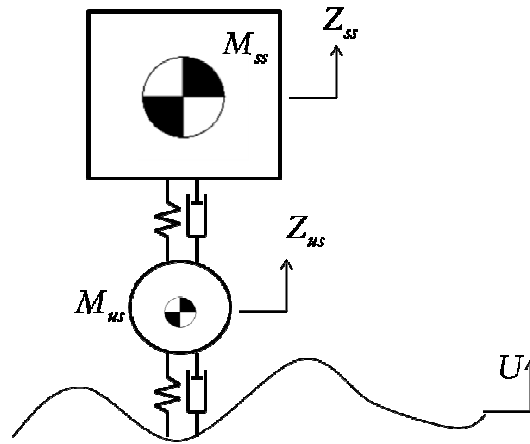


**Figure 3. Schematic of pitch plane model with eight degrees of freedom.**

Linearised air spring models and nonlinear leaf spring suspension models were simulated (Cebon 1999; Fox et al. 2007). Based on design trends in the US trucking fleet observed by Hajek and Billing, the tandem suspensions of the pitch plane models were assumed to be 90% air, 10% leaf suspensions for the tractor, and 70% air, 30% leaf suspensions for the trailer (Hajek, Billing 2002). The steer axles were assumed to all be leaf sprung (Hajek, Billing 2002). A constant damping factor was assumed for the air suspensions. The value was calculated from the laden mean of the tandem axle weight distribution. Further information on all parameters and the leaf spring model can be found in (Goodrum, Cebon 2010).

## 2.4. Quarter Car Models

Cole and Cebon noted that Quarter Car Models (QCMs) with suitable parameter values can capture the important features of the frequency content of heavy vehicle axle dynamics (Cebon 1985; Cole, Cebon 1995; Cebon 1999). Gillespie also stated that QCMs are better for representing the dynamics of heavy vehicles than for passenger cars (Gillespie 1992). The equations of motion for a two degree-of-freedom QCM were given in detail by Cebon, and Figure 4 is a schematic of a representative QCM (Cebon 1985, 1999).



**Figure 4. Schematic of Quarter Car Model with degrees of freedom labelled.**

QCMs are computationally more efficient than pitch plane models and have been used to model traffic in several previous studies of pavement damage (Collop et al. 1996; Costanzi, Cebon 2007; Belay et al. 2008). QCMs were included here because they provide a good compromise between the accuracy of dynamic tyre forces and increased computational speed.

## 3. Modelling Spatial Repeatability

### 3.1. Spatial Repeatability

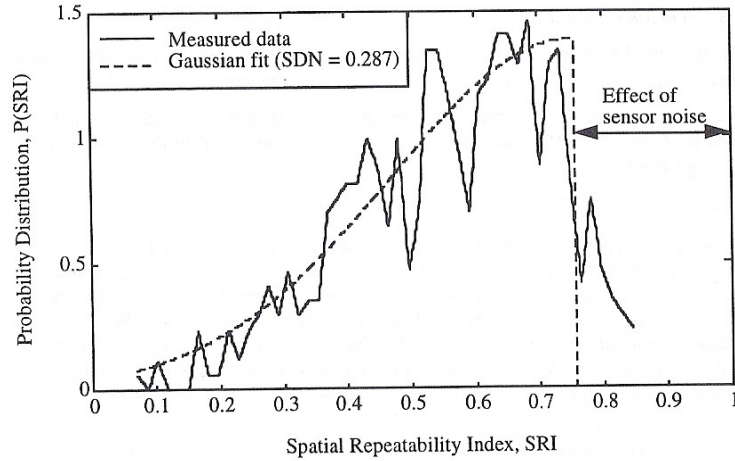
Spatial repeatability arises because trucks are similar in weights, dimensions, and dynamic characteristics and travel at similar speeds. As a result, each vehicle will apply its peak forces at approximately the same places along the pavement surface (Cole, Cebon 1992).

Cole et al. defined the spatial repeatability index (SRI) as the correlation coefficient between a dynamic tyre force histories,

$$SRI = \frac{E[(x(t) - m_x)(y(t) - m_y)]}{\sigma_x \sigma_y} \quad (1)$$

where  $x$  and  $y$  are dynamic tyre forces histories,  
 $m_x$  and  $m_y$  are the mean forces of  $x$  and  $y$ , respectively,  
and  $\sigma_x$  and  $\sigma_y$  are the standard deviations of  $x$  and  $y$ , respectively (Cole, Cebon 1992; Cole et al. 1996).

Collop applied spatial repeatability to pavement modelling with his whole-life pavement performance modelling methodology (Collop, Cebon 1995; Collop et al. 1996). Figure 5 is a plot of a Gaussian probability distribution fit to the SRI data measured on a highway using a 'load-measuring-mat' by (Cole et al. 1996).



**Figure 5. Gaussian probability distribution fit to measured spatial repeatability index (SRI) data (from Cebon) (Cebon 1999).**

Collop assumed that all dynamic tyre force histories generated by the vehicle fleet had equal magnitude but differing phases in the Fourier domain (Collop et al. 1996). Therefore, Equation (1) collapses to the following,

$$SRI = \cos(\varphi) \quad (2)$$

where  $\varphi$  is the phase difference between tyre force histories in radians (Collop et al. 1996).

Collop used a Gaussian probability distribution to fit the measured data in Figure 5, giving:

$$P(\varphi) = \frac{1}{\sqrt{2\pi}\sigma_{SRI}} e^{-\frac{(\cos(\varphi)-1)^2}{2\sigma_{SRI}^2}}, \quad (3)$$

where  $P(\varphi)$  is the probability of a given phase angle,  $\varphi$ , and  $\sigma_{SRI}$  is the standard deviation of the Gaussian probability distribution.

$\sigma_{SRI}$  was previously called the ‘Spatial Distribution Number’ (SDN), as indicated in Figure 5.

Collop developed a method to synthesise dynamic tyre forces using the Gaussian probability distribution given by Equation (3) (Collop et al. 1996). He used discrete phase angles from this known distribution to create a spatially repeatable fleet of QCMs (Collop et al. 1996). First, Collop generated a golden dynamic tyre force history with one QCM. Second, this golden tyre force history was passed through a Fast Fourier Transform. Finally, Collop phase shifted the golden history by each constant phase angle to generate a set of tyre force histories with the same statistical repeatability as the measured reference distribution. Further justification of this method can be found in (Collop et al. 1996; Collop, Cebon 1997).

An alternate measure of spatial repeatability, suggested by Cole, is the fleet normalised aggregate tyre force,

$$NAF_{fleet} = \frac{\sum_{j=1}^{N_V} \sum_{k=1}^m F_{i,jk}}{A_{fleet}} \quad (4)$$

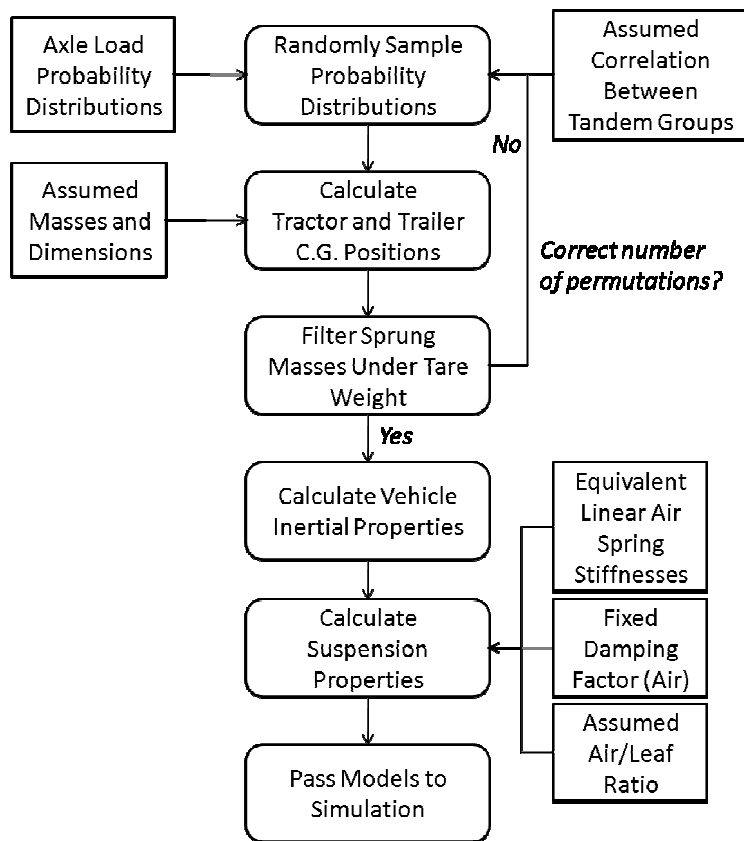
where,  $F_{i,jk}$  is the tyre force at the  $i$ -th road point due to the  $k$ -th axle of the  $j$ -th vehicle,  $m$  is the number of axles on each vehicle,  $N_V$  is the total number of vehicles, and  $\overline{A_{fleet}}$  is the mean of the double sum in the numerator (Cole et al. 1996).

The fleet normalised aggregate force gives a spatial (time-domain) picture of the cumulative pattern of traffic loading.

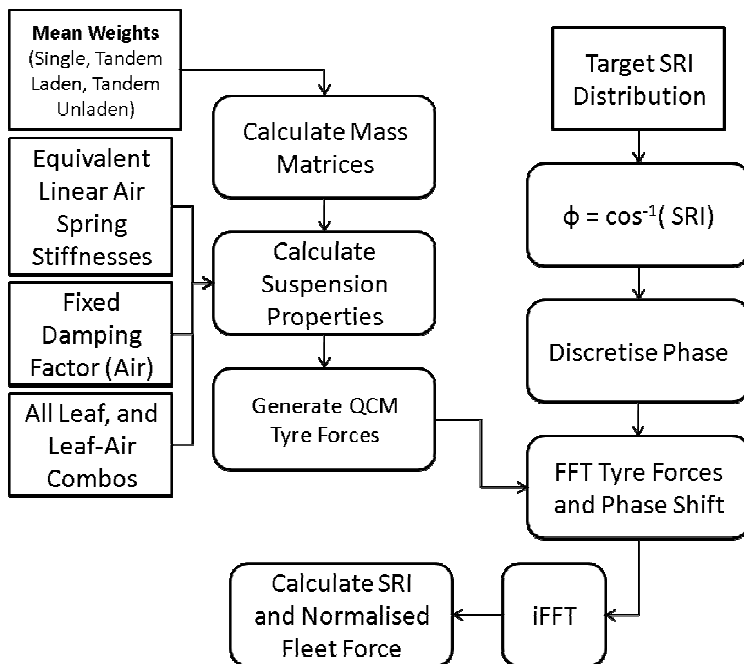
### 3.2. Method

The analytical approach taken in this project was as follows:

1. A ‘reference’ fleet of several-thousand vehicle models was generated using detailed, ‘per-vehicle’ WIM data (containing static loads, axle spacings, and vehicle speeds). Some unknown vehicle parameters were randomised to make the set of models representative of the dynamic characteristics of the traffic stream.
2. The ‘reference’ fleet was used to generate:
  - a. Axle load probability distributions – the expected starting point for pavement design using the ME-PDG
  - b. A simulation of the dynamic loads generated by all of the vehicles over a simulated length of road. Statistical information about the spatial repeatability of this ‘reference’ fleet was calculated from these loads.
3. The axle load probability distributions from (2a) were randomly sampled to create three alternate methods of synthesising the loading of a fleet of heavy vehicles
  - a. A ‘target’ fleet of pitch-plane models (as in Figure 3). This fleet was considered to be representative of the ‘best possible’ conditions available for pavement design using the input expected for the ME-PDG. The process of generating this fleet is outlined in Figure 6.
  - b. A fleet of QCMs (as in Figure 4) generated in a similar fashion to the pitch-plane models of (3a), and simulated in groups of three for efficiency.
  - c. A fleet of phase-shifted QCMs following Collop’s example. The process of generating this fleet is outlined in Figure 7. This method used the SRI statistics from the ‘target’ fleet in (3a)
4. A convergence study was performed to determine the minimum number of vehicles required to characterise the measured ‘per-vehicle’ dataset. It was found that 7000 vehicles for the ‘reference’, and 1000 vehicles for the ‘target’ and randomised QCMs, respectively, achieved convergence. 20 discrete phases captured the salient features of the ‘target’ SRI statistics for the phase-shifted QCM method. Details of the convergence study can be found in (Goodrum, Cebon 2010).
5. The statistics of the spatial repeatability generated by each of the three methods was compared with the ‘reference’ values from (2b) to quantify the accuracy of each calculation. This statistical comparison and the computation time were used to draw conclusions about the best approach for synthesising spatially repeatable traffic loading in a manner suitable for ‘whole-life performance’ calculations.



**Figure 6. Flow diagram of the randomised pitch-plane model generation methodology**



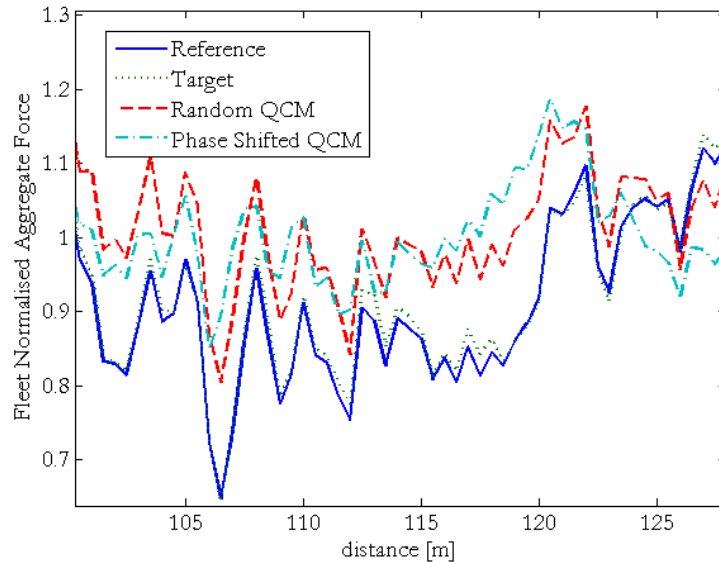
**Figure 7. Flow diagram outlining the QCM phase shifting method. The ‘target’ SRI curve input was generated using the method shown in Figure 6**



## 4. Results & Discussion

### 4.1. Normalised Aggregate Force and Fourth-Power Forces

Figure 8 is a comparative plot of the fleet normalised aggregate forces generated by the four simulation methods.

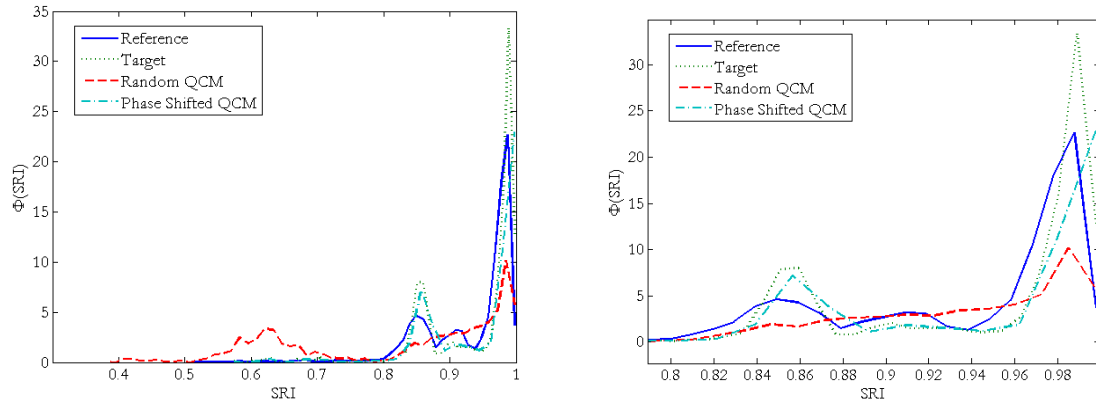


**Figure 8. Comparison of fleet normalised aggregate force histories for each of the four simulated vehicle fleets**

The ‘reference’ and ‘target’ time histories agree very well, while the time histories from the phase shifted and randomised QCM fleets appear to follow a similar pattern. This comes down to the inherent dynamics of each model; the ‘reference’ and ‘target’ histories were generated by pitch-plane models, the randomised QCMs and phase-shifted models used QCMs that lack pitch interaction and wheelbase filtering. This lack of pitch dynamics is believed to be the reason for the wider spread of the randomised QCM SRI curve, discussed in Section 4.2, below. Although the phase-shifted QCMs and ‘target’ fleet have different time histories, their repeatability statistics (given below) are similar.

### 4.2. SRI Curves

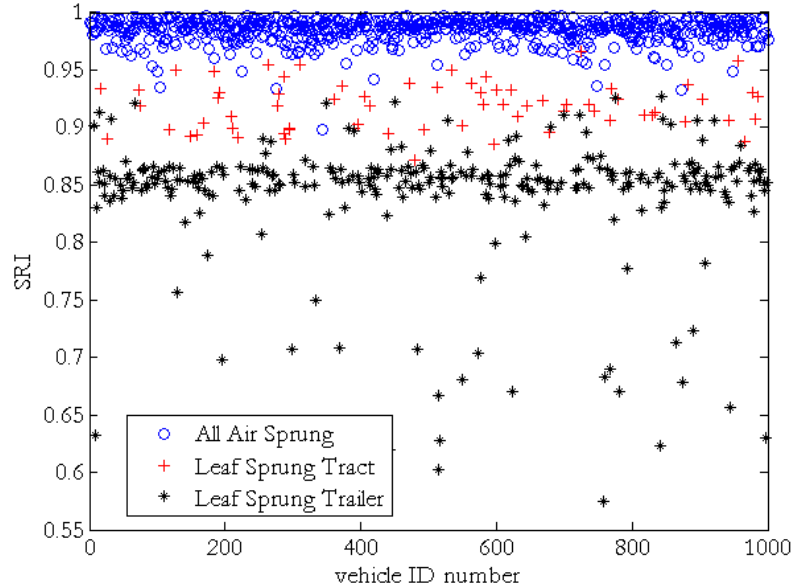
Figures 9a and b are comparative plots of the SRI curves generated by each of the four simulated fleets and a re-plotting of the region  $0.8 < \text{SRI} < 1$ , respectively. From Figures 9, it can be seen that the ‘target’ and the phase-shifted SRI curves agree well, as expected. The ‘target’ curve is narrower than the ‘reference’ curve because the axle spacings for each ‘target’ vehicle were assumed constant. All curves show a sharp peak close to the maximum value of one, with secondary peaks in the region of  $\text{SRI}=0.85$ .



**Figures 9a and 9b. Comparison plot of SRI curves for each of the four simulation methods (left); the same comparison replotted for the range  $0.8 < \text{SRI} < 1$  (right)**

Figure 10 is the ‘target’ SRI data from Figure 9a re-plot versus vehicle reference number. The data markers in the plot show which vehicles are entirely air sprung, have leaf-sprung tractors, and leaf-sprung trailers. From this data, it is evident that vehicles with similar suspension characteristics correlate well with each other, as expected.

In the SRI curve for the randomised QCMs, another, broader peak is evident in the neighbourhood of  $\text{SRI} = 0.6$ . A similar divide to that observed in Figure 10 was found between the all air sprung QCMs and the QCM groupings with at least one leaf sprung axle (Goodrum, Cebon 2010). As for the pitch-plane models, so can it also be concluded that suspension type has a dominant effect on the correlation of QCM dynamic tyre forces.



**Figure 10. Plot of ‘target’ SRI data (from Figure 9) versus vehicle reference (ID) number showing the scatter of suspension types in the fleet**

### 4.3. Computation Time and Summary of Results

Table 1 provides a summary of computation time and accuracy metrics for each simulation method.

Method	Simulation Time (One Run) [sec]	Simulation Time (20 years, @ 1 per week) [days]	R <sup>2</sup> of SRI Curves Relative to 'Reference'	R <sup>2</sup> of SRI Curves Relative to 'Target'	Correlation of ATFs to 'Reference' Fleet	Correlation of ATFs to 'Target' Fleet
Reference	147600	1777	1	-	1	-
Target	26000	313	0.91	1	0.99	1
Random QCM	24000	290	0.76	0.76	0.75	0.74
Phase Shifted QCM	90	1.5	0.53	0.89	0.52	0.48

**Table 1. Summary of simulation time, goodness-of-fit, and correlation of normalised aggregate forces for all four simulated fleets**

The large sample size required for the 'reference' fleet increased the simulation time significantly over the other three methods. Since the randomised QCMs were simulated in groups of three, the total time saved over the 'target' pitch-plane models was small.

Although the phase-shifted method includes the overhead time of generating the 'target' SRI curve first (i.e., from measured data or more realistic simulations), this is a 'one-time' cost and the phase-shifted models still represent a significant decrease in computation time. For example, if one was simulating 20 years worth of traffic in weekly intervals, 1040 separate traffic calculations would be required. Using the randomised pitch-plane models of the 'target fleet' would take approximately one month of CPU time. Conversely, the phase-shifted method would require only 1.5 days of CPU time, accounting for the overhead of generating the 'target' SRI distribution; a 95% reduction in computing time.

Goodness-of-fit was calculated for the SRI curves to quantify the similarity (or difference) between results. The pitch-plane SRI curves (Reference and Target) agreed well with one another. SRI values for the randomised QCMs were calculated by aggregating the QCM tyre forces as if they were axle forces. It is believed that the randomised QCMs were less correlated due to the lack of pitch interaction. Differences in suspension characteristics play a more dominant role for the QCMs than in the pitch-plane models, where pitch behaviour and wheelbase filtering are important, as well.

The normalised aggregate forces for the 'reference' fleet correlate with the other time histories in a similar fashion to the SRI curves of Section 4.2. Although the SRI curve for the phase-shifted QCMs is close to the SRI curve of the 'target' fleet, the pattern of loading in the time domain was different (Figure 8). The SRI is a measure of the correlation of the dynamic tyre forces of the fleet with respect to one time history within the same fleet. Conversely, the normalised aggregate force is the sum of all tyre forces at points along the road. A high value of SRI does not ensure a high value of aggregate force; it does, however, indicate the similarity in phase in the time-domain seen in Figure 8. Given the substantial computational benefit of the phase-shifting method, and the excellent agreement of the SRI statistics, it is believed that the phase-shifted QCMs are still the best available method for simulating dynamic tyre forces for whole-life pavement performance calculations in which the effects of millions of axle loads need to be simulated over the lifetime of the road surface.

## 5. Conclusions

1. Pitch dynamics have a significant effect on the aggregate tyre forces and spatial repeatability statistics of vehicles with both pitch and bounce modes of vibration. For vehicles with only bounce modes, it was found that the suspension type has a dominant effect on spatial repeatability.
2. The fleet aggregate tyre force histories of four simulated fleets comprising pitch-plane models, quarter car models, randomly sampled, and phase-shifted models showed good agreement in phase, with correlation coefficients all greater than 0.5. Differences in the magnitude of the aggregate tyre force histories can be attributed to differences in (or lack of) pitch behaviour.
3. The phase shifting method is the most computationally efficient means of simulating traffic for whole-life pavement performance calculations of road surface life, requiring only 4-5% of the computation time of the more realistic pitch-plane models. However, the increased speed comes at the cost of accuracy in the time-domain.

## 6. Acknowledgements

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