IMPLEMENTATION OF ACTIVE STEERING ON A MULTIPLE TRAILER LONG COMBINATION VEHICLE



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Abstract

In order to introduce longer heavy vehicles with multiple articulation joints between vehicle units into the UK and other European countries, rear steering of the vehicle units is required to allow for sufficient manoeuvrability. This paper details the development and testing of trailer steering controllers for forward and reverse travel of a B-double vehicle. Practical implementation of the controllers on the Cambridge Vehicle Dynamics Consortium's experimental Long Combination Vehicle (LCV) is outlined, and test results of the system's low speed forward and reversing performance are shown.

Keywords: Heavy vehicles, long combination vehicles, LCV, b-double, active steering, rear steering, trailer steering.

1. Introduction

Long combination vehicles (LCVs), with two or more trailers, offer substantial benefits over tractor semi-trailer vehicles: reducing traffic congestion, shipping costs and fuel consumption (CO₂ emissions), while also reducing road wear (Woodrooffe and Ash, 2001). LCVs are common in countries with long, straight roads such as Australia, Canada, and the USA, but they are not approved for widespread use in Europe. Figure 1 (a) and (b) show examples of vehicles currently allowed on UK roads.



Figure 1: (a) and (b) Maximum sizes of vehicle currently allowed in the United Kingdom. (c) Experimental concept vehicle.

One of the barriers to the introduction of LCVs in Europe is their poor low speed manoeuvrability, both in the forward and reverse directions. In Europe, normally only vehicles that are able to negotiate a 'standard' roundabout (12.5m external radius and 5.3m inner radius in the UK) are allowed on the roads. This manoeuvrability restriction is likely to remain even if longer vehicles were introduced.

Figure 2 (a) shows a simulation of an unsteered b-double attempting the UK roundabout manoeuvre. The rear trailer cuts across the inner circle, and eventually begins to reverse. Having multiple articulation joints causes instability in reverse motion, making manual reversing manoeuvres a highly skilled, or even impossible task. Delivery locations may therefore need to be redesigned to accommodate these less manoeuvrable vehicles, at significant cost.



Figure 2: Simulation results for driving a B-double vehicle around a UK standard minimum size roundabout, (a) unsteered, (b) steered.

In order to address the problems of manoeuvrability of LCVs in both the forward and reverse directions, the use of 'active' (i.e. computer controlled) trailer steering systems has been proposed (Jujnovich and Cebon, 2008). Figure 2 (b) shows a simulation of a steered b-double negotiating the UK roundabout manoeuvre, showing a significant improvement in performance over the unsteered case in Figure 2 (a).

Some theoretical studies have already been carried out on multi-trailer vehicle steering, systems for forward motion including: Rangavajhula's work on a multiply-steered triple trailer vehicle using LQR optimal control (Rangavajhula and Tsao, 2007), which showed significantly improved off-tracking of trailers in simulations; De Bruin also simulated lateral guidance of multiply-articulated buses using steered axles (de Bruin, Damen et al., 2000). Both of these studies show significant benefits available from steering of trailers, but neither were tested in experiments. Some commercial interest has also been shown in using trailer steering for multi-trailer vehicles (Denby, 2010; Trackaxle, 2010), culminating in prototype vehicles using technology already proven on tractor semi-trailer vehicles.

Commercial steering systems already exist for semi-trailers using mechanical or hydraulic actuation either passively or computer controlled (VSE, 2005; Trackaxle, 2010; Tridec, 2010). These systems commonly use the simple 'command steer' strategy (explained later), which gives benefits for steady state off-tracking and tyre wear, but causes tail-swing (Sweatman, Coleman et al., 2003; Jujnovich and Cebon, 2008) and reduces high speed yaw stability. Following work done by Hata and Notsu (Hata, Hasegawa et al., 1989; Notsu, Takahashi et al., 1991), Jujnovich was able to improve on this performance with his CT-AT (conventional tractor – active trailer) controller strategy (Jujnovich and Cebon, 2008), which was implemented on an actively steered tractor semi-trailer vehicle (Jujnovich, Roebuck et al., 2008). Using this strategy, the path of the rear of the semi-trailer accurately followed that of tractor hitch, eliminating steady state off-tracking and tail-swing. The aim of this paper is to extend this tractor semi-trailer CT-AT controller work for use on an LCV.

Reversing of a vehicle and trailer has been the subject of much research, but the majority of work concerns steering the tractor front axle only, to assist in stabilising the vehicle yaw motion (Sordalen and Wichlund, 1993; Halgamuge, Runkler et al., 1994; Tilbury, Sordalen et al., 1995; Tanaka, Taniguchi et al., 1999; Altafini, Speranzon et al., 2001; Rajamani, Zhu et al., 2003; Zimic and Mraz, 2006; Novak, Dovzan et al., 2008; Pradalier and Usher, 2008). This approach can also assist in reversing of multi-trailer vehicles, (e.g. truck with dolly semitrailer) (Tilbury, Sordalen et al., 1995; Tanaka, Taniguchi et al., 1999; Altafini, Speranzon et al., 2001), but the lack of trailer steering limits the improvements it can make to vehicle manoeuvrability. Kimborough (Kimbrough and Chiu, 1990; Kimbrough, Chiu et al., 1990; Chiu and Kimbrough, 1991) developed controllers for the reversing of steered trailers, using the trailer steering to stabilise the trailer vaw motion (hitch angle). However, the fact that the driver's input must control the tractor steer axle directly, still limits the vehicle manoeuvrability to some extent. Commercial steered trailers (VSE, 2005; Trackaxle, 2010) tend to be equipped with a manual over-ride of trailer steer angle to allow the trailer steering to be controlled remotely by a second person during low speed manoeuvres. However, this generally requires a second operator, rather than being automatic. Unlike these commercial systems, a vehicle with computer controlled tractor and trailer steering gives a unique capability to implement the CT-AT strategy in reverse, allowing automatic control of all trailer axles with only a single control input i.e. only one driver is needed.



Figure 3: The Cambridge Vehicle Dynamics Consortium's test vehicle (configured as 'Bdouble'). fitted with active rear steering hardware on the two axle groups

This paper presents the results of testing the new multiple-trailer forward and reversing versions of the CT-AT strategy, designated CT-AT-AT. This papers concentrates specifically on the development and implementation of these strategies on an experimental 'B-double' combination as shown in Figure 1(c) and Figure 3.

2. Vehicle Design

Testing of steering strategies was carried out using the Cambridge Vehicle Dynamics Consortium (CVDC) steered B-double vehicle, shown in Figure 3. All trailer axles are actively steered and computer controlled (note that the front axle on the b-trailer is a conventional lift axle – consequently only two of the three axles on this trailer are steered), with steering actuators specially designed for the vehicle (see section 2.2). The rearmost semi-trailer unit is the original 12.5m tri-axle, fully steered CVDC trailer as reported in previous work (called the A-trailer here) (Jujnovich, Roebuck et al., 2008). The 'link' trailer (called the B-trailer) was designed and built for this research programme. The overall length was chosen as 13.4m (carrying a 9.6m shipping container, with 11.0m between hitches), the maximum that could negotiate the standard UK roundabout manoeuvre with perfect path following, given the steer angle limit of its axles.

2.1 Active Steering Axle

Figure 4 shows an active steering axle used on the test vehicle. The steering hardware consists of: (i) A steering axle with cambered king-pins and modified conventional steering linkage; (ii) A hydraulic cylinder containing a novel locking mechanism which acts to secure the actuator in the straight ahead position when not active; (iii) A reservoir of pressurised hydraulic fluid (not shown) that is used for emergencies to drive the actuator back to the central position if either the external hydraulic power supply or the external electrical supply ceases. Under these conditions a separate hydro-mechanical valve is used to control the actuator back to the central position, where it is locked; (iv) An electrically-powered hydraulic power supply, mounted on the vehicle frame above the axles.



Figure 4: CAD view from below of the B-trailer steering axle with the 'four-bar' steering mechanism and actuator position has been overlaid.

2.2 Instrumentation and control

The overall signal and control architecture is shown in Figure 5, including the controllers and sensors fitted to the test vehicle. The active steering control system consists of 3 levels of controller: A proprietary C-based controller on each vehicle unit performs closed loop control of axle steer angles and interfaces with sensors (i.e. performs both the axle controller (AC) and trailer controller (TC) functions while a Matlab XPC based 'Vehicle controller' (VC) generates steer angle demands for the whole vehicle based on sensor inputs from the TCs, communicated via CAN.



Figure 5: Signal and control architecture.

The vehicle safety system allows complete or partial system shutdown in case of failures of sensors, actuators, or the vehicle not behaving as expected. Power shutdown triggers steering actuators to center and lock. The system can allow one trailer to remain steering (if it is safe to do so) to avoid incapacitating the vehicle on a roundabout. Video cameras record lines on

road, allowing measurement of the offset of tractor 5th wheel, b-trailer hitch, and rear doors. Other vehicle sensors measure tractor speed, trailer and tractor steer angles, articulation angles, as well as yaw, roll and pitch rates and acceleration in 3 axes. Only a limited number of these sensors are used for active control, the rest are used for performance monitoring.

3. Controller Design

For forward travel, two controllers were compared: the conventional "command steer" controller, and the CT-AT controller (Jujnovich, Roebuck et al., 2008), modified for use in a multi-trailer vehicle i.e. CT-AT-AT. For reversing, an AT-AT-CT type controller was developed.

3.1 Command Steer Controller for Forwards Travel

The conventional command steer algorithm steers each axle in proportion to the articulation angle between the trailer and the unit in front, such that all axles turn about a common center (Jujnovich, Roebuck et al., 2008). The trailer then behaves as if it has an unsteered axle part way down the vehicle as shown in Figure 6 (a) . The location of this axle is a free design parameter (the "effective length" of the trailer), and was chosen to be half the distance between the hitch points in order to give no off-tracking of the hitch points at the two ends of the trailer in the steady state. For the multi-trailer case, each trailer can act independently using only its own articulation angle sensor, but needs to be programmed with certain parameters about the trailer in front (e.g. its effective length, and the location of its hitch), to compensate for the hitch velocity not being parallel to the longitudinal axis of the trailer in front. This strategy is simple to implement, requiring a minimal sensor set (2 articulation angle sensors), but does have limitations. Because the strategy is based on constant radius turning, its performance during transient curvatures is compromised, leading to tail-swing on entry to curves, and additional cut-in on exit.



Figure 6: (a) Diagram showing the command steer strategy as applied to a b-double travelling in the forwards direction. (b) Diagram showing the CT-AT-AT strategy as applied to a b-double travelling in the forwards direction, and AT-AT-CT in reverse.

3.2 CT-AT-AT Control For Forwards Travel

The CT-AT-AT controller is based on Jujnovich (Jujnovich, Roebuck et al., 2008) CT-AT semi-trailer path-following controller (Jujnovich and Cebon, 2008). This controller aims to achieve perfect path following with all trailers, such that the two 'follow points' (B-hitch and the rear doors of rear trailer) follow the lead point: the tractor hitch (shown in Figure 6 (b)). This eliminates tail-swing and cut-in, as well as minimising lateral tyre forces.

The CT-AT-AT vehicle controller requires two more sensors than the command steer algorithm (tractor speed and steer angle), though these are commonly available. Any saturation of the b-trailer steering actuators (i.e. reaching maximum steer angle of the wheels) results in off-tracking of both trailers, but the vehicle will asymptote back to perfect path tracking when the steering actuator(s) unsaturate. Data needs to be passed between individual trailers in order to correct for the rear-steer angle of the trailer in front, and to share the tractor speed sensor data (though the latter could be eliminated by using speed sensors on each unit). The trailers of the CT-AT-AT controller are therefore not truly stand-alone, but rely on being hitched to a vehicle that can supply this extra sensor information. Note that this information transfer can be omitted with little loss of performance for the special case of hitching a trailer close to an unsteered axle of the unit in front, (because articulation angle then gives a good estimate of the lead point heading angle). This special case applied to Jujnovich (Jujnovich, Roebuck et al., 2008), i.e. a steered semi-trailer attached to a conventional tractor unit. But it could not be applied to the rear trailer of the b-double studied here, because its link trailer is steered.

3.3 CT-AT-AT Control For Reversing

The reversing controller applies a similar path following strategy, but with lead and follow -points swapped. The driver controls the steer angle of the a-trailer axles using a joystick and camera on the rear of vehicle. The lead point is therefore the midpoint of the rear doors of the trailer. The b-trailer steer angles are then controlled to achieve perfect tracking of the path of the rear door with the B hitch. The tractor steer angles are controlled so that the tractor hitch follows the B-hitch (as shown in Figure 6 (b)). In a commercial system, the tractor steering would be computer-controlled using a servo motor. For testing this arrangement was not available, so the CT-AT-AT reversing controller passed steer angle demands to the 'driver' (via a VDU) – effectively using the driver as a servo.

The steering on A and B trailers fairly directly influences heading of the lead point on that unit, but tractor steering has much more indirect effect on heading of tractor hitch (tractor articulation angle is equivalent to the steering angle of the A or B trailer). Control of the tractor hitch position was therefore expected to be less accurate than control of the B-hitch position.

This reversing controller requires only two extra components over the forward CT-AT-AT controller: the joystick and reversing camera. The most significant barrier to the adoption of this controller is the provision of tractor 'angle overlay' front axle steering, however such systems already exist for cars, and there seems to be no technical barrier preventing similar systems being developed for trucks. Currently, saturation of the steering on A-trailer violates the controller assumptions, leading to unrecoverable off-tracking of the other follow points. Further work is therefore required before the algorithm could be used commercially. The approach used by Jujnovich, using a non-saturating model-matching controller, is thought to be viable.

4. Results

4.1 Forward travel, Command Steer and CT-AT-AT.

The command steer and CT-AT-AT controller for forwards travel were tested on the standard UK roundabout manoeuvre shown in figure 5 (a). The vehicle must negotiate the roundabout without crossing the 12.5m or 5.3m circles (Anon, 1997). For testing, the center of the tractor front axle followed an 11.25m radius (dashed line in figure 5 (a)) and exited after 450degrees.



Figure 7: (a) Standard UK roundabout (Anon, 1997), as used for the test manouvre in the forward travel direction. Dashed line indicates target path of front axle center. (b) 'tear drop' manouvre used to evaluate the reversing control strategies.

Command steer:

Figure 8 shows deviation from the path of the tractor front axle (the dashed line in figure 5 (a)) of the tractor hitch, b-trailer hitch, and A-trailer rear doors, measured using the path tracking cameras. During the steady state (distance 50-80m), the b-hitch and A-trailer rear doors followed the tractor hitch within 0.2m. However, this steady state did not develop until the trailer had travelled a significant distance into the manoeuvre (\sim 30m from roundabout entrance).



Figure 8: Experimental results showing the offset (relative to the ideal path) of various points along the length of a b-double, steered using the command steer strategy, whilst travelling round a UK minimum radius roundabout.

The hitch of the B-trailer swung out on entrance to manoeuvre by 0.5m (as shown in Table 1) due to the short effective length, which led to a large effective rear overhang. The tail of the A-trailer then swung out even further (1.6m) when it reached the entry point. The A-trailer tail-swing can be seen in Figure 9(a) - the left hand white line is the outside of the roundabout entry lane. This behaviour is particularly hazardous as the tail-swing is into the driver's blind spot. Upon exiting the manoeuvre, both trailers then cut in further than their steady state (up to 2.52m for the rear trailer as shown in Table 1. Finally, the A-trailer took some distance (~10m) to settle back to the path of the tractor and b-hitch (known as the Exit Settling Distance).

Table 1: Summary of performance measures for forward steering strategies tested.

	Command Steer	CT-AT-AT
Cut in (m)	2.52	1.1
Tail swing (m)	1.65	0.0
Exit settling distance (m)	10	0.0



Figure 9: Images showing rear of steered b-trailer when entering UK minimum size roundabout when using (a) command steer strategy and (b) CT-AT-AT strategy to control the steered trailer axles.

CT-AT-AT strategy

Figure 10 shows off-tracking vs. distance for the vehicle operating the CT-AT-AT path following controller. Throughout the manoeuvre, the B-hitch followed the tractor 5^{th} wheel closely (within ~0.3m), and the A-trailer rear doors followed the B-hitch (within ~0.4m), as intended. The large tail-swing visible for the command steer vehicle was therefore eliminated for both trailers (see photo in Figure 9(b)), and cut-in was also limited to that of the tractor 5^{th}

wheel at 1.1m as shown in Table 1. Exit settling distance was also eliminated (see Table 1). The small error in A-trailer rear door positioning in the steady state is due to the build-up of sensor errors from trailer to trailer, but does not cause the trailer rear doors to sweep any path not already covered by the tractor unit, so is unlikely to cause unexpected collisions.



Figure 10: Experimental results showing the offset (relative to the ideal path) of various points along the length of a b-double, steered using the CT-AT-AT strategy, whilst travelling round a UK standard roundabout (Figure 7 (a)).

4.2 Experimental Results: Reversing

To test the reversing algorithm, a 'Teardrop' manoeuvre was followed as shown in Figure 7 (b). The manoeuvre was designed to be similar to a roundabout, but with larger radius to give some steer angle 'overhead' for use in correcting for any path errors which developed.

Figure 11 shows the path error at the rear doors (lead point), B-trailer hitch and tractor hitch vs. distance during the teardrop manoeuvre test. The entry and exit points of the 12.5m radius section are shown on the figure. The path following of the rear doors is controlled by the driver using the joystick, and achieves path error less than 1.1m throughout manoeuvre. This performance was limited by the frame rate and preview provided by the reversing camera, and could certainly be improved by a more suitable camera. The B-trailer hitch followed the path of the rear doors very closely, deviating by less than 0.4m from the lead point path throughout the test.

The tractor hitch suffered a much greater path error than the b-hitch (the 1.5m lateral range of the camera was exceeded at two points). The first deviation occurred on entry to the curve, where there was a step change in path curvature, which requires a step change in the tractor articulation angle. Time-lag in the development of this articulation angle led to path error at the hitch. The second error was due to overshoot at the point where the rear doors developed 1.1m error (a driver path following inaccuracy). Simulations suggest that this performance

could be improved if the tractor steering was computer controlled, as this would reduce the control lag. Also, a more sophisticated algorithm for controlling the tractor steering could improve this performance e.g. using a preview controller. Derivation of this controller will be carried out as further work.



Figure 11: Experimental results showing the offset (relative to the ideal path) of various points along the length of a b-double, steered using the AT-AT-CT strategy, whilst travelling in reverse around the tear drop manouvre.

5. Conclusions

- 1. A B-double vehicle with five steered trailer axles was built to test steering control algorithms in forward and reverse directions.
- 2. Forwards travel:
 - (i) The CT-AT-AT path following controller was developed by extending the CT-AT strategy to multi-trailer vehicles.
 - (ii) The B-trailer vehicle path tracking performance was tested on the standard UK roundabout manoeuvre.
 - (iii) Steering both trailers using the conventional 'Command Steer' strategy gave acceptable steady state off-tracking, but excessive (2.52m) tail-swing on entry to the manoeuvre.
 - (iv) The CT-AT-AT strategy improved off-tracking without creating tail-swing, following the 5th wheel of the tractor within 0.7m for both trailers.
- 3. Reversing
 - (i) The CT-AT-AT controller was implemented in the reverse direction, steered by the driver using a joystick and a camera mounted at the rear of the vehicle.
 - (ii) The path following capability of the vehicle was successfully demonstrated by reversing the vehicle around a 'teardrop' manoeuvre.

- (iii) Error from the path of the rear doors was less than 0.4m for the B-trailer hitch, but much greater (>1.5m) for the tractor hitch.
- (iv) Further work is required to reduce the tractor hitch off-tracking by using more sophisticated control strategies, and by automatic control of the tractor steer axle.

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